(A1) [known problem, discussed in lecture]:

thermal occupation:

$$\varrho \overset{①}{=} \left(1 + e^{-\frac{\Delta E}{k_B T}}\right)^{-1} \begin{pmatrix} 1 & 0 \\ 0 & \exp\left(-\frac{\Delta E}{kT}\right) \end{pmatrix}$$

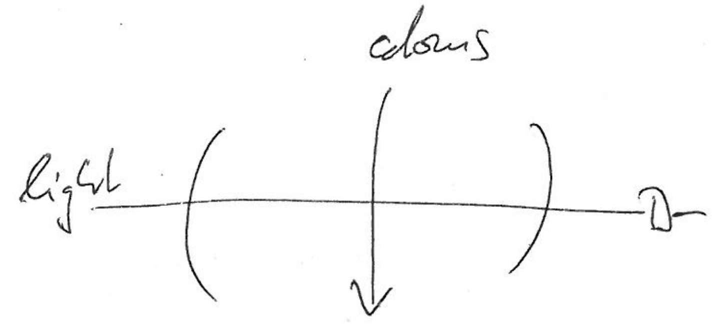
with $\Delta E = E_2 - E_1$

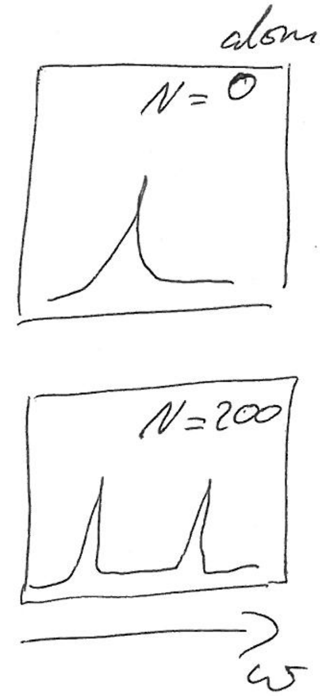
general two-level system with N_1, N_2:

$$\varrho \overset{①}{=} \begin{pmatrix} N_1/N_0 & 0 \\ 0 & N_2/N_0 \end{pmatrix}$$

- decoherence: $\varrho_{12}, \varrho_{21} \rightarrow 0$ ①

A2) [known, similar question in PS]





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- **strong coupling**: atom cavity rate g_0 needs to be larger than cavity decay rate. 1

- angular frequency:
  $$\omega_0 = 2\pi \frac{c}{\lambda} = 3.2 \cdot 10^{15} \ \frac{\text{rad}}{s}$$ 1

- vacuum Rabi splitting:
  $$\Delta \Omega^{vac} = 2\sqrt{N}\, g_0 = 1.8 \cdot 10^{8} \ \frac{\text{rad}}{s}$$ 1

A3 a phase of matter (Bosons), where all palides are described by the same wavefunction. all particles (atoms, Bosons) occupy the same quantum state collectively, needs critical temperature

1 and density to occure.

$$f(T) = 0.5 = 1 - \left(T/T_c\right)^3 \quad 1$$

$$\left(T/T_c\right)^3 = 0.5$$

$$\Rightarrow T = T_c (0.5)^{1/3} = 123\ nK \quad 1$$

(A4) [similar question on PS]

Calculate the Q of the cavity : $Q = \frac{\omega}{\Delta\omega}$ (1)

in high Q regime ($\Delta\omega$ small), we can write :

$$\frac{\omega}{\Delta\omega} \simeq \frac{\lambda}{\Delta\lambda}$$ (1)

$$\Rightarrow Q = \frac{\lambda}{\Delta\lambda} = \frac{930\ nm}{0{,}18\ nm} = 5200$$ (1)

Purcell factor :

$$F_p = \frac{3Q\left(\frac{\lambda}{n}\right)^3}{4\pi^2 V_0}$$ (1)

$$= \frac{3 \cdot 5200\,(9.3 \cdot 10^{-7}/3.5)^3}{4\pi^2\,(1.8 \cdot 10^{-18})} = 4.1$$ (1)

Result means $F_p > 1 \Rightarrow$ cavity enhances the spontaneous emission rate of q-dot.

(1)

(A5) [based on PS question]

Photon flux from laser is ~~gib~~ given by:

$$\Phi = \frac{P}{\hbar\omega} \quad (1)$$

$$= \frac{10^{-13}\,W}{2{,}41\ eV} = 2{,}59 \cdot 10^{5}\ \frac{\text{photons}}{\text{sec}}$$

(1)

The average photon count is then given by: (in time interval T)

$$N(T) = \eta\,\Phi\,T \quad (1)$$

$$= 0{,}1 \cdot 2{,}59 \cdot 10^{5} \cdot 0.5$$

$$(= 12950 \text{ photons})$$

(1)

$$= 13\,000 \text{ photons}$$

B1 [book work, seen in lecture]

(a) Calculate the average photon flux,

-) photon energy of 1.17 eV 1

$$\bar{n} = \frac{P}{\hbar\omega} = \frac{300\ \mathrm{W}}{1.17\ \mathrm{eV}} = 1.6 \cdot 10^{21} \frac{\text{photons}}{\text{s}}$$

1 1

-) uncertainty of photon number:

$$\Delta n = \left(|\alpha| + 1/4\right)^2 - \left(|\alpha| - 1/4\right)^2$$

$$= |\alpha| = \sqrt{\bar{n}}$$ 1

$$= 4.0 \cdot 10^{10}$$ 1

-) assumption: all classical noise has been eliminated, phase uncertainty is given by: $\Delta\phi\, \Delta n \geq \frac{1}{2}$ 1

$$\Rightarrow \Delta\phi = \frac{1}{2}\Delta n = 1.3 \cdot 10^{-11}\ \text{radians}$$

1

B1) (b) to see fringe shift due to displacement δL, we need:

$$\frac{\delta L}{\lambda} > \frac{\Delta \phi}{2\pi} \qquad 1$$

$\Delta\phi = 1{,}3 \cdot 10^{-12}$ radians

$\lambda = 1064\,\text{nm}$

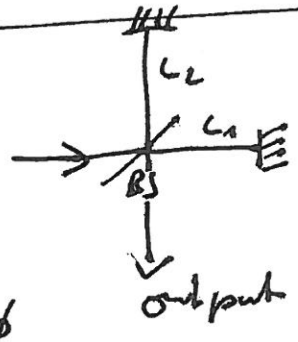
$$\Rightarrow \quad \delta L = 2.2 \cdot 10^{-19}\,\text{m} \qquad 1$$

(c) strain is equal to the fractional length change divided by the original length. With cavity enhancement: 1

$$\text{strain} = \frac{\delta L}{50 L} = \frac{2.2 \cdot 10^{-18}\,\text{m}}{50 \cdot 4\,\text{km}} = 1.1 \cdot 10^{-23}$$

1

(d) 1



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$$E^{out} = E_1 + E_2$$

$$= \tfrac{1}{2} E_0 e^{i2kL_1} + \tfrac{1}{2} E_0 e^{i2kL_2} e^{i\Delta\phi} \qquad 1$$

$$= \tfrac{1}{2} E_0 e^{i2kL_1}\left(1 + e^{i2k\Delta L} e^{i\Delta\phi}\right) \qquad 1$$

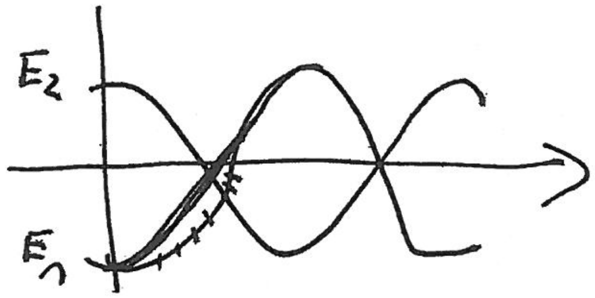
(see addition next page!)

B7) (c) $\Delta \phi = \pi$

$\Rightarrow \phi_1^r = \phi_2^r + \pi$ 1

$\cos(\phi_2^r) + \cos(\phi_2^r + \pi) = 0$ 1

$\Rightarrow$ sum of both is always 0 ! 1



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(d) addition:

condition for field maxima:

$$\frac{4\pi}{\lambda} \Delta L + \Delta \phi = 2m\pi \quad (m \in \mathbb{Z})$$ 1

B2) (a) $g^{(2)}(\tau) \overset{1}{=} \dfrac{\langle I(t)\, I(t+\tau)\rangle}{\langle I(t)\rangle \langle I(t+\tau)\rangle}$, $g^{(1)}$ is for E-field correlations 1

(b) if $\tau \gg \tau_c$ intensity fluctuations will be completely uncorrelated with each other at time t and $(t+\tau)$.

discussed in lecture

In general: $$I(t) = \langle I \rangle + \Delta I(t)$$

$$\langle I(t)\, I(t+\tau)\rangle_{\tau \gg \tau_c} =$$

$$\overset{1}{=} \Big\langle \big(\langle I \rangle + \Delta I(t)\big)\big(\langle I \rangle + \Delta I(t+\tau)\big) \Big\rangle$$

$$\overset{1}{=} \langle I \rangle^2 + \langle I \rangle \langle \Delta I(t) \rangle + \langle I \rangle \langle \Delta I(t+\tau) \rangle + \langle \Delta I(t)\, \Delta I(t+\tau) \rangle$$

$$= \langle I \rangle^2$$

used that: 1

$\langle \Delta I(t) \rangle = 0$ and

1

$\langle \Delta I(t)\, \Delta I(t+\tau) \rangle_{\tau \gg \tau_c} = 0$

as complete un-correlation means a random change of sign with time and therefore averages to zero.

B2) b) continued ...

$$\Rightarrow \quad g^{(2)}(\tau \gg \tau_c) = \frac{\langle I(t)\, I(t+\tau)\rangle}{\langle I(t)\rangle^2}$$

1

$$= \frac{\langle I(t)\rangle^2}{\langle I(t)\rangle^2} = 1$$

---

c) if $\tau = 0 \Rightarrow g^{(2)}(0) = \frac{\langle I(t)^2\rangle}{\langle I(t)\rangle^2}$ 1

$$\left.\begin{aligned} g^{(2)}(0) &\geq 1 \\ g^{(2)}(0) &\geq g^{(2)}(\tau) \end{aligned}\right\}$$ 1, if one of the relations is stated.

-) perfectly coherent light has time-independent intensity $I_0 \Rightarrow$ 1

$$g^{(2)}(\tau) = \frac{\langle I(t)\, I(t+\tau)\rangle}{\langle I(t)\rangle^2} = \frac{I_0^2}{I_0^2} = 1$$

1

for all τ.

B2) c) continued...

·) a chaotic light source with time-varying intensity:

$$\Rightarrow \quad \langle I(t)^2 \rangle > \langle I(t) \rangle^2 \qquad 1$$

$$\Rightarrow \quad g^{(2)}(0) = \frac{\langle I(t)^2 \rangle}{\langle I(t) \rangle^2} > 1 \qquad 1$$

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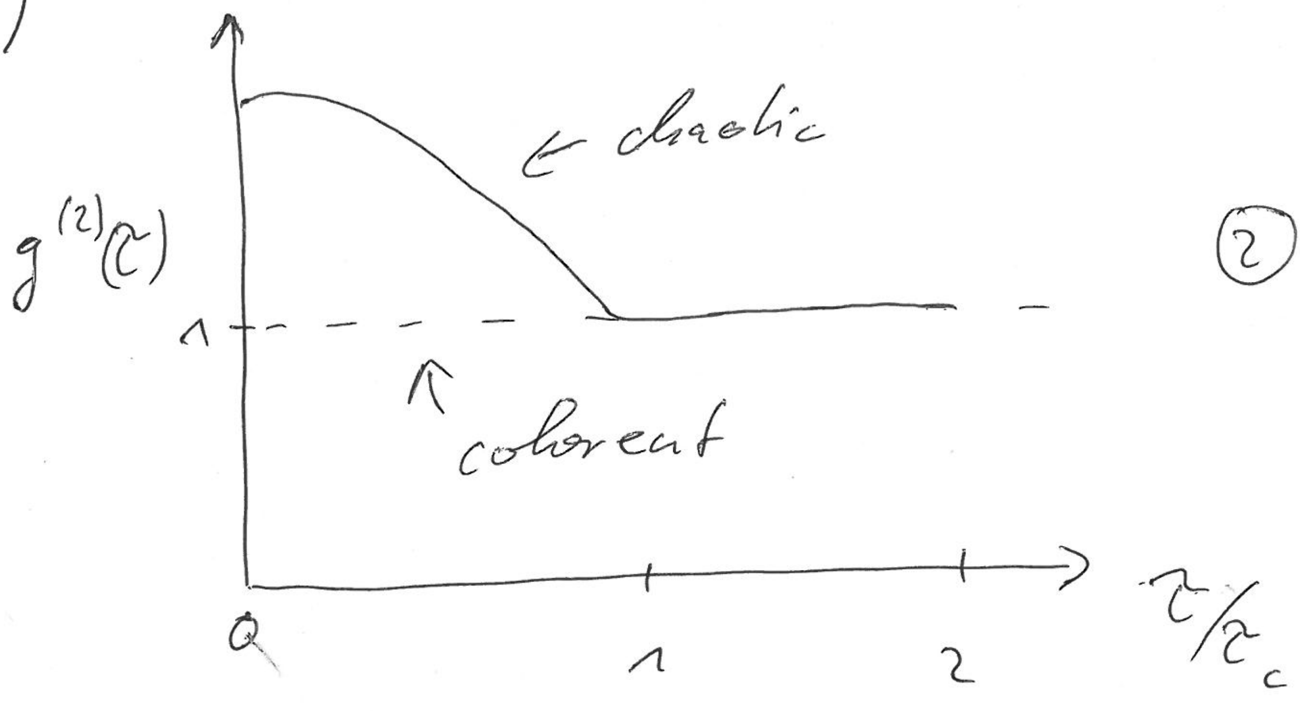
d)



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B2) e)

$$g^{(2)}(0) = \frac{\langle I(t)^2 \rangle}{\langle I(t) \rangle^2} = \frac{\langle I_0^2 (1 + A \sin(\omega t))^2 \rangle}{I_0^2}$$

$$= \langle (1 + A \sin(\omega t))^2 \rangle \qquad 1$$

[we used $\langle I(t) \rangle = I_0 \langle (1 + A \sin(\omega t)) \rangle$
$= I_0$, since $\langle \sin(\omega t) \rangle = 0$.

compute time average by integration over time interval T: ($T \gg \frac{1}{\omega}$)

$$g^{(2)}(0) = \frac{1}{T} \int_0^T (1 + A \sin \omega t)^2 \, dt \qquad 1$$

$$= \frac{1}{T} \int_0^T (1 + 2A \sin \omega t + A^2 \sin^2 \omega t) \, dt \qquad 1$$

[using $2 \sin^2 x = (1 - \cos 2x)$ and with both $\sin \omega t$ and $\cos 2\omega t$ averaging to zero]

$$\ldots \quad g^{(2)}(0) = 1 + \frac{A^2}{2T} \int_0^T (1 - \cos 2\omega t) \, dt$$

$$= 1 + \frac{A^2}{2} \qquad 1$$

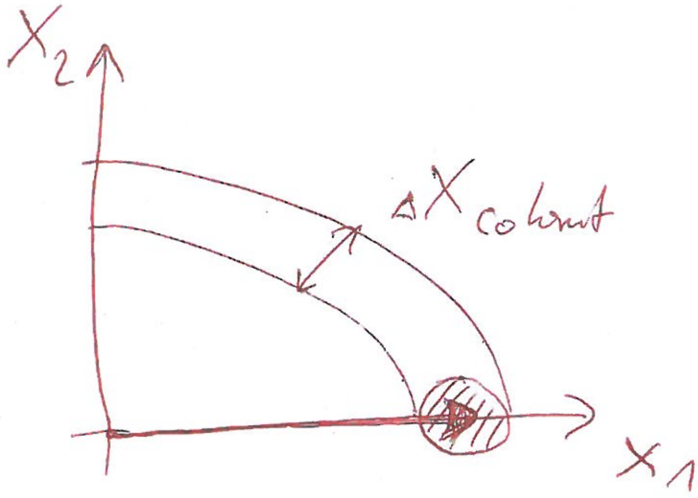
B 2) e) ... continued ...

$\rightarrow$ $g^{(2)}(0)$ is always greater than unity and the maximum value is 1.5 (for $|A| = 1$). (1)

discussed in lecture / based on problem sheet

B3) (a) The electric field amplitude is directly proportional to the length of the
1 vector representing the state in the phasor diagram. A state will exhibit amplitude squeezing if the radial
1 uncertainty, namely ΔX_{cohrnt}, is equal to 1/2 in dimensionless quadrature units.

Consider coherent state:

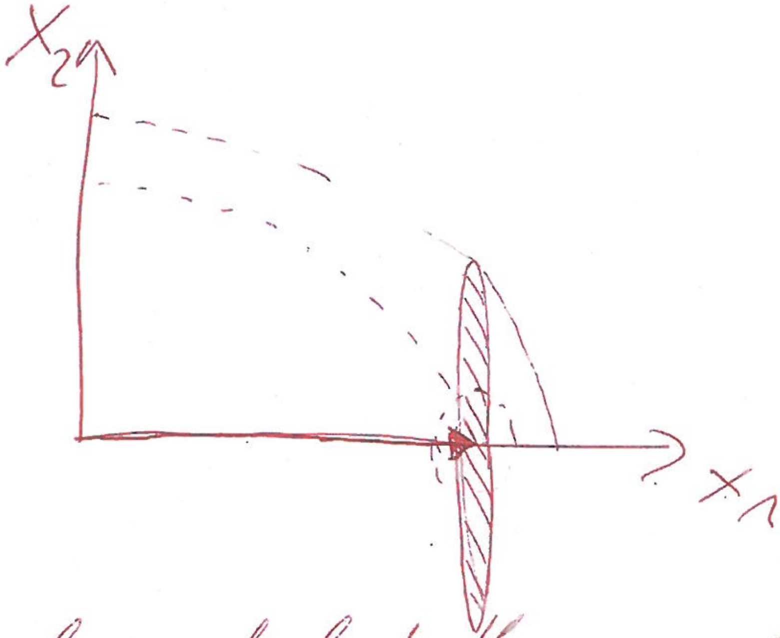
1 

The radial uncertainty (ΔX_{cohrnt}) is equal to 1/2 (in quadrature units)

B(3) (a) ... continued ...

Now the quadrature - squeezed state has elliptical uncertainty area:

(1)



and has at least the same area as the coherent state.

The radial uncertainty (~~variation~~ uncertainty of phasor length as indicated by dashed lines) is

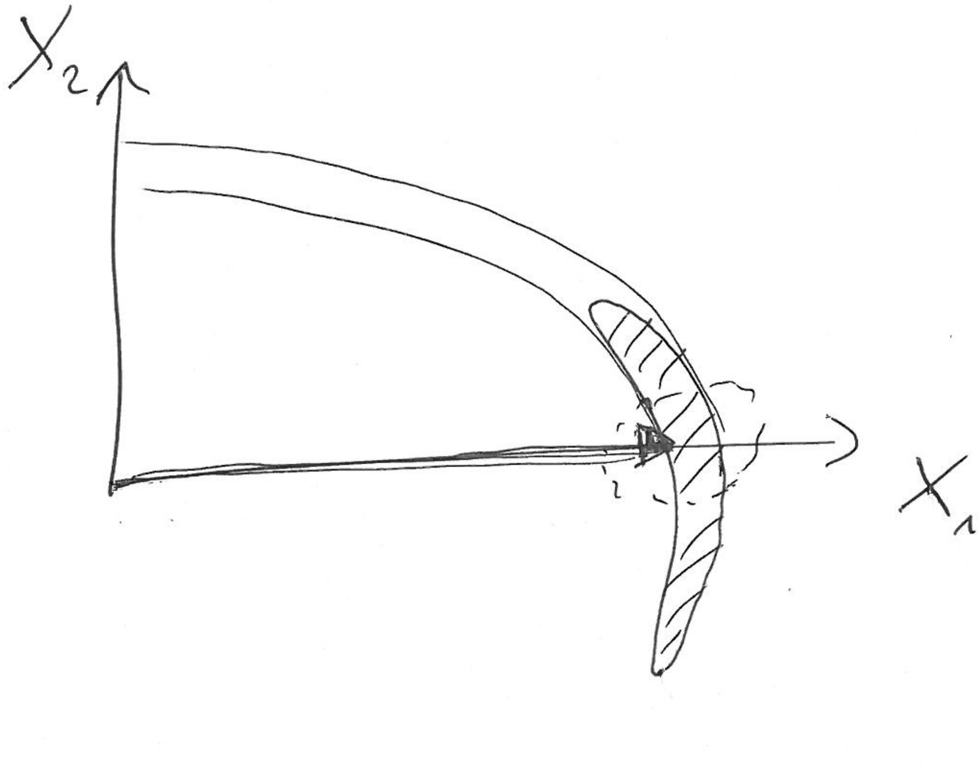
(1) minimized, when axes of ellipse are aligned with those of the phasor.

$\Rightarrow$ for strong squeezing the ellipse gets more and more elongated

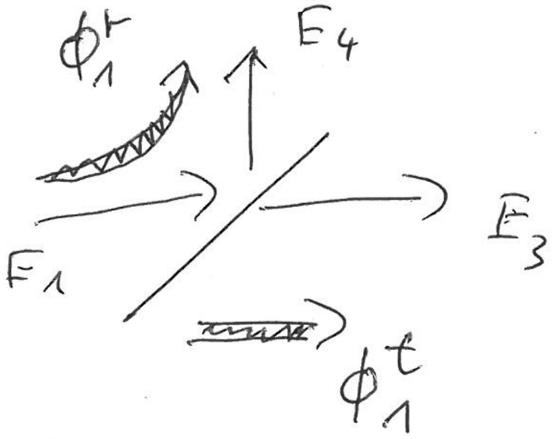
(1) $\Rightarrow$ radial uncertainty becomes larger than $\Delta X_{\text{coherent}}$ $\Rightarrow$ no amplitude squeezing.

B3) b) To get strong amplitude squeezing it is necessary to distort the uncertainty area in such a way as to reduce the radial variations. This means that the
1 uncertainty area has to be shaped like a banana with the radial uncertainty minimized at the expense of the increased angular uncertainty.

1



B3) c) phase shifts for input fields E_1, E_2:



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$$\boxed{\begin{aligned} E_3 &= E_1 t_1 e^{i\phi_1^t} \\ E_4 &= E_1 r_1 e^{i\phi_1^r} \end{aligned}}$$

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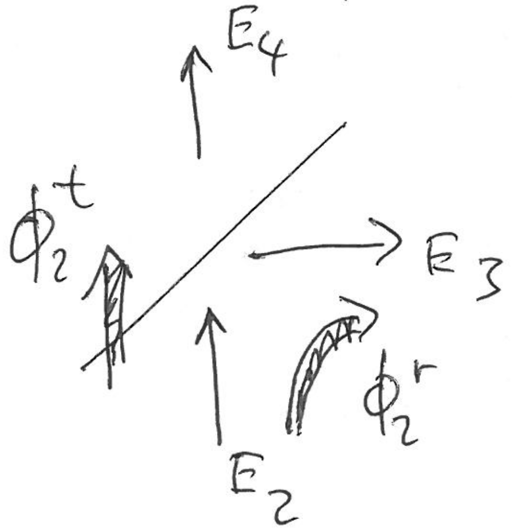

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$$\boxed{\begin{aligned} E_3 &= E_2 r_2 e^{i\phi_2^r} \\ E_4 &= E_2 t_2 e^{i\phi_2^t} \end{aligned}}$$

1

which makes in total:

$$\Rightarrow \quad E_3 = E_1 t_1 e^{i\phi_1^t} + E_2 r_2 e^{i\phi_2^r}$$

$$E_4 = E_1 r_1 e^{i\phi_1^r} + E_2 t_2 e^{i\phi_2^t}$$

1

B2) c) ... continued...

→ for 50:50 BS the power reflection and transmission coefficients are both equal to 1/2. Power is proportional to square

1 of amplitude ⟹

1 $r_1 = t_1 = r_2 = t_2 = \frac{1}{\sqrt{2}}$

$$E_3 = \frac{1}{\sqrt{2}}\left(E_1 e^{i\phi_1^t} + E_2 e^{i\phi_2^r}\right)$$

⟹

1 $$E_4 = \frac{1}{\sqrt{2}}\left(E_1 e^{i\phi_1^r} + E_2 e^{i\phi_2^t}\right)$$

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d) $E_3 E_3^* + E_4 E_4^* =$

$$= \frac{1}{2}\left(E_1 e^{i\phi_1^t} + E_2 e^{i\phi_2^r}\right)\left(E_1 e^{-i\phi_1^t} + E_2 e^{-i\phi_2^r}\right)$$

1 $$+ \frac{1}{2}\left(E_1 e^{i\phi_1^r} + E_2 e^{i\phi_2^t}\right)\left(E_1 e^{-i\phi_1^r} + E_2 e^{-i\phi_2^t}\right)$$

$= \ldots$

B3) d) continued ...

$$E_3 E_3^* + E_4 E_4^* = \frac{1}{2}\left(E_1^2 + E_2^2 + E_1 E_2\left(e^{i(\phi_2^r - \phi_1^t)} + e^{i(\phi_1^t - \phi_2^r)}\right)\right)$$

(1)

$$+ \frac{1}{2}\left(E_1^2 + E_2^2 + E_1 E_2\left(e^{i(\phi_1^r - \phi_2^t)} + e^{i(\phi_2^t - \phi_1^r)}\right)\right)$$

(1)

$$= E_1^2 + E_2^2 + E_1 E_2\left(\cos(\phi_2^r - \phi_1^r) + \cos(\phi_1^r - \phi_2^t)\right)$$

$\Rightarrow$ power conservation: $E_3 E_3^* + E_4 E_4^* = E_1^2 + E_2^2$

(1) $\overset{E_1, E_2 \neq 0}{\Longrightarrow}$ $\cos(\phi_2^r - \phi_1^t) + \cos(\phi_1^r - \phi_2^t) = 0$ ∎

B4) (a) $E_0(t) = E_{peak}\, e^{-t^2/\tau^2}$ (given in question)

$$\Rightarrow I(t) = I_0\, e^{-\frac{2t^2}{\tau^2}} \qquad 1$$

find τ at pulse width of $\frac{1}{2} I_0$:

$$I(t_{1/2})/I_0 \overset{1}{=} e^{-(2t_{1/2}^2/\tau^2)} = 0.5 \qquad 1$$

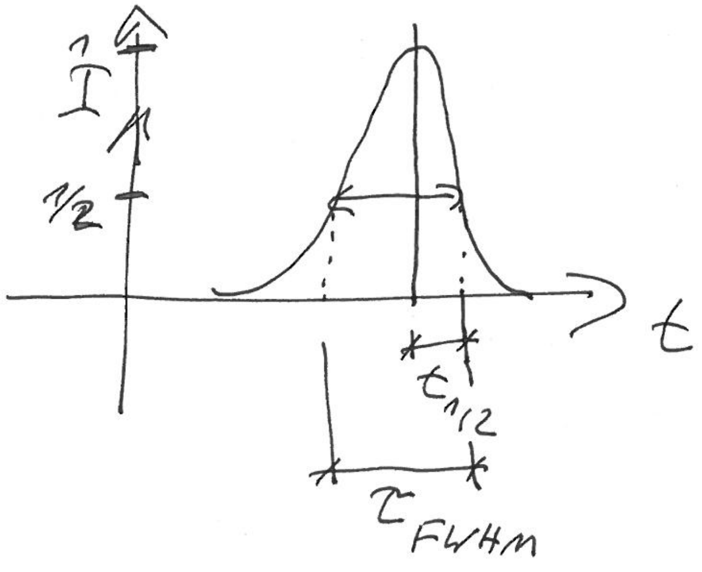
(ln)

$$\Rightarrow t_{1/2} = \sqrt{\frac{\ln 2}{2}}\,\tau \qquad 1$$

$$\rightarrow \tau_{FWHM} = 2\cdot t_{1/2} = 2\sqrt{\frac{\ln 2}{2}}\,\tau \qquad 1$$

$$= 1.777\,\tau$$

$\Rightarrow$ with $\tau_{FWHM} = 1ps \rightarrow \tau = 0{,}85\,ps$



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B4) b) pulse area:

$$\Theta = \frac{\mu_{12}}{\hbar} \int_{-\infty}^{+\infty} E_0(t)\, dt$$ (given in question)

$$= \frac{\mu_{12}}{\hbar} E_{peak} \int_{-\infty}^{\infty} e^{-\frac{t^2}{\tau^2}}\, dt$$ (1)

$$= \sqrt{\pi}\, \mu_{12} E_{peak} \frac{\tau}{\hbar}$$ (1)

$$\Rightarrow \quad E_{peak} = \frac{\Theta \hbar}{\sqrt{\pi}\, \mu_{12} \tau}$$

with $\Theta = \frac{\pi}{2}$

$\mu_{12} = 10^{-29}\,Cm$

$\tau = 0{,}85\,ps$

$$= 11 \frac{MV}{m}$$

(1)

B4) c)

$$E_{pulse} = A \int_{-\infty}^{+\infty} I(t)\, dt \qquad (1)$$

refractive index $n = 1$ and with Gaussian beam:

$$E_{pulse} = \frac{1}{2} A c \varepsilon_0 \int_{-\infty}^{+\infty} E_0(t)^2\, dt \qquad (1)$$

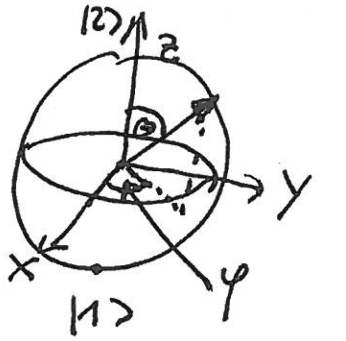
$$= \frac{1}{2} A c \varepsilon_0 E_{peak}^2 \int_{-\infty}^{+\infty} e^{-\frac{2t^2}{\tau^2}}\, dt \qquad (1)$$

$$= \sqrt{\frac{\pi}{8}}\, A c \varepsilon_0 E_{peak}^2 \tau \qquad (1)$$

-) area of laser beam $A = \pi \left(10^{-6}\,m\right)^2 = 3.1 \cdot 10^{-12}\, m^2$ (1)

$\Rightarrow E_{pulse} = 0{,}53\ \mu J$ (1)

B4) (d)



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→ azimuthal angle for rotation is arbitary

→ choose rotation in the $\varphi = 0$ plane.

→ <u>initial position</u>: state vector points downwards with $\Theta = \pi$ 1

→ after the $\pi/2$-pulse we arrive at

$(\Theta, \varphi) = (\pi/2, 0)$ 1

$|\psi\rangle = c_1|1\rangle + c_2|2\rangle$

→ we know: $c_2 = e^{i\varphi}\cos(\Theta/2)$
$c_1 = \sin(\Theta/2)$

$\Rightarrow c_1 = 1/\sqrt{2}$
$c_2 = 1/\sqrt{2}$ 1

$$\Rightarrow \quad |\psi\rangle = \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{\sqrt{2}}|2\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)$$

1