SEMESTER 1 EXAMINATION 2014-2015

Coherent Light & Coherent Matter

Duration: 120 MINS (2 hours)

*Answer **all** questions in **Section A** and **only two** questions in **Section B**.*

***Section A** carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.*

***Section B** carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.*

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

Section A

A1. Write down the density matrix for a thermal ensemble of two-level atoms at temperature T. What is the general definition of the density matrix for two-level systems with N_1 atoms in the lower state and N_2 atoms in the upper state? What happens to the off-diagonal elements of the density matrix if an initially coherent superposition of two states decoheres? [3]

A2. A cavity of modal volume $1.9 \cdot 10^{-11} m^3$ is tuned resonant to one sodium hyperfine transition at 589nm with transition dipole moment $\mu_{12} = 2.1 \cdot 10^{-29} Cm$. Draw the scheme of the experiment. How is strong coupling defined? Calculate the angular frequency of the transition and the angular frequency of the vacuum Rabi splitting for a cavity containing 200 atoms and with a coupling content of $g_0 = 6.3 \cdot 10^{-6}$ rad/s. [4]

A3. Describe the key features of a Bose-Einstein condensate. Find the temperature at which more than half of the atoms are in the 3d condensate, if the critical temperature to form the condensate T_c is 154nK. [3]

A4. A quantum dot emitting at 930 nm is placed at the centre of a resonant micropillar cavity containing material of refractive index 3.5 (GaAs). The modal volume is 1.8 $\times$ 10^{-18} m^3 and the spectral width $\Delta\lambda$ of the resonant mode is 0.18 nm. Calculate the Purcell factor. What is the physical significance of this result? [6]

A5. An attenuated light beam from an Ar laser operating at 514 nm (2.41 eV) with a power of 0.1 pW is detected by a photo-counting system of quantum efficiency 10$\%$, with the time interval set at 0.5 s. Calculate the average number of photons counted. [4]

Section B

B1. The LIGO gravitational wave interferometer is basically a Michelson interferometer with a 50:50 beam splitter. The relative displacement δL of mirrors M_1 and M_2 (attached to arms of length L) due to gravitational waves will be detected. The operation wavelength is 1064nm and the light power is 300W. We are interested in the sensitivity of LIGO.

(a) Working with the number-phase uncertainty relation, what is the phase uncertainty of the light? [7]

(b) What is the minimum displacement δL that can be detected? [2]

(c) Calculate the minimum strain that can be detected for an interferometer arm length $L = 4km$. Assume that the light is traveling 50 times along L. [2]

(d) Write down the output electric field for a Michelson interferometer with a 50:50 beam splitter and a general phase factor $\Delta\phi$. Include an expression for the relative length of the arms $\Delta L = L_2 - L_1$. Assume that the input field consists of parallel rays from a linearly polarized monochromatic source of wavelength λ (and $k = 2\pi/\lambda$) and amplitude E_0. Draw the setup. Where do the field maxima in the interference pattern occur ? [5]

(e) For a beam splitter with two input beams, show the relation:

$$\cos(\phi_2^r) + \cos(\phi_1^r) = 0,$$

is fulfilled in the general case that the relative phase shift between the reflection of the two input beams $\Delta\phi = \pi$, with ϕ_1^r and ϕ_2^r being the phases due to reflection of the two input beams on the beam splitter. Draw the two cosine functions. [4]

B2. This question is about the second order correlation function used to quantify the temporal coherence of classical light. Consider a light source with constant average intensity such that $\langle I(t)\rangle = \langle I(t+\tau)\rangle$ and for which the time-scale of the intensity fluctuations is determined by the coherence time τ_c.

(a) Write down the definition of the second order correlation function $g^{(2)}(\tau)$. How is it different from the first order correlation function $g^{(1)}(\tau)$? [2]

(b) Show that in general $g^{(2)}(\tau) = 1$, for $\tau \gg \tau_c$. [Hint: Start with the general expression for the intensity: $I(t) = \langle I\rangle + \Delta I(t)$ and substitute into $g^{(2)}(\tau)$.] [5]

(c) What general statement can you make about $g^{(2)}(\tau)$ when $\tau = 0$ for classical light? Write down $g^{(2)}(\tau)$ when $\tau = 0$ for both perfectly coherent light and chaotic light. [6]

(d) Draw a diagram of the dependence of $g^{(2)}$ on τ for chaotic and for perfectly coherent light. [2]

(e) Evaluate $g^{(2)}(0)$ for a monochromatic light wave with a sinusoidal intensity modulation such that $I(t) = I_0(1 + A\ \sin(\omega t))$, with $|A| \leq 1$. In this case is $g^{(2)}(0)$ always smaller or greater than unity? [5]

B3. This question is about squeezed states of light and classical waves at a 50:50 beam splitter.

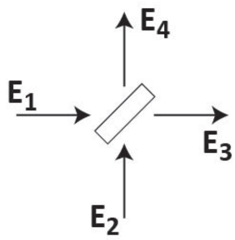

Figure 1: Input and output field on a 50:50 beam splitter.

(a) Explain what amplitude squeezing is and how it is represented by a phasor. Why will light with very strong quadrature squeezing not exhibit amplitude squeezing, no matter how the axes of the uncertainty ellipse are chosen? Use quadrature uncertainty diagrams for a coherent state and strongly quadrature-squeezed light as part of your argument. [6]

(b) Draw and explain why strongly amplitude-squeezed light has an uncertainty area shaped like a banana on a quadrature uncertainty diagram. [2]

(c) Squeezed light can be analysed by homodyne detection using a beam splitter. Consider now classical waves and a 50:50 beam splitter as shown in Fig.1. Let the phase shifts on the transmission and reflection be written ϕ_i^t and ϕ_i^r, respectively, where $i = 1, 2$. Assume that E_1 and E_2 are real. Draw diagrams for the possible phase shifts on transmission and reflection for both input fields and verify that the total output fields must be in the form:

$$E_3 = \frac{1}{\sqrt{2}}[E_1 \exp(i\phi_1^t) + E_2 \exp(i\phi_2^r)],$$
$$E_4 = \frac{1}{\sqrt{2}}[E_1 \exp(i\phi_1^r) + E_2 \exp(i\phi_2^t)].$$

[8]

(d) Consider the same situation as in (c) and, by consideration of conservation of energy, show that:

$$\cos(\phi_2^r - \phi_1^t) + \cos(\phi_1^r - \phi_2^t) = 0.$$

[Hints: The input fields are real. Start by considering the quantity $(E_3E_3^* + E_4E_4^*)$. Then use the fact that energy conservation means that the total input power must be equal to the total output power.] [4]

B4. A pulsed laser beam is focused to a spot of radius 1.00μm on a gas of atoms with a dipole moment of $\mu_{12} = 1.00 \times 10^{-29}$Cm at the laser frequency to drive Rabi oscillations between a resonant atomic two-level system. The FWHM duration of the laser pulse is τ_{FWHM}=1.00ps.

(a) Derive a formula for the pulse duration τ when the intensity of the pulse is dropped to half its peak value. Hint: Start by using a Gaussian field for the laser: $E_0(t) = E_{peak} \exp(-t^2/\tau^2)$ and calculate the intensity at FWHM. [6]

(b) Derive a formula for the peak field of the laser pulse E_{peak} using the formula for the pulse area $\Theta = \frac{\mu_{12}}{\hbar} \int_{-\inf}^{+\inf} E_0(t)dt$. Calculate E_{peak} for a pulse area of $\Theta = \pi/2$. [3]

(c) Now calculate the pulse energy E_{pulse} required to rotate the Bloch vector representing the state of the atomic two-level system by $\pi/2$ radians for Gaussian pulses with a duration (FWHM) of $1ps$. Hint: The pulse energy is the product of the area of the beam A times the integral over the laser intensity. [6]

(d) If the system is initially in the ground state, find the state of the system at the end of the pulse. Draw the Bloch sphere. [5]

END OF PAPER