UNIVERSITY OF SOUTHAMPTON PHYS6012W1

SEMESTER 1 EXAMINATION 2012/13

Coherent Light, Coherent Matter

Duration: 120 MINS

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*Answer **all** questions in **Section A** and **only two** questions in **Section B**.*

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

A Sheet of Physical Constants will be provided with this examination paper.

An outline marking scheme is shown in brackets to the right of each question.

Only university approved calculators may be used.

Section A

A1. Calculate the coherence time for the 589.0nm line of a sodium lamp operating at 100° C if the line is Doppler-broadened. The mass of sodium is 23amu. [4]

A2. Write down the density matrix for a thermal ensemble of two-level atoms at temperature T. What is the general definition of the density matrix for two-level systems with N_1 atoms in the lower state and N_2 atoms in the upper state? What happens to the off-diagonal elements of the density matrix if an initially coherent superposition of two states decoheres? [3]

A3. A source emits a regular train of pulses, each containing exactly two photons. State and explain the value of $g^{(2)}(0)$. Show that $g^{(2)}(0) = 1$ for a coherent state. Explain how amplitude squeezed light can help to reduce noise in some systems below shot noise. Further explain further why strongly amplitude squeezed light would have a uncertainty area line shaped like a banana. [6]

A4. A cavity of modal volume $1.9 \cdot 10^{-11} m^3$ is tuned resonant to one sodium hyperfine transition at 589nm with transition dipole moment $\mu_{12} = 2.1 \cdot 10^{-29} Cm$. Draw the scheme of the experiment. Calculate the vacuum Rabi splitting frequency for a cavity containing 200 atoms. [4]

A5. Evaluate the Bose-Einstein condensation temperature for 10,000 ^{87}Rb atoms in a trap of angular frequency 10^3rad/s, and find the temperature at which more than half of the atoms are in the condensate. [3]

Section B

B1. The LIGO gravitational wave interferometer is basically a Michelson interferometer with a 50:50 beam splitter. The relative displacement δL of mirrors M_1 and M_2 (attached to arms of length L) due to gravitational waves will be detected. The operation wavelength is 1064nm and the light power is 300W. We are interested in the sensitivity of LIGO.

(a) Working with the number-phase uncertainty relation, what is the phase uncertainty of the light? [7]

(b) What is the minimum displacement δL that can be detected? [2]

(c) Calculate the minimum strain that can be detected for an interferometer arm length $L = 4km$. Assume that the light is traveling 50 times along L. [2]

(d) Write down the output electric field for a Michelson interferometer with a 50:50 beam splitter and a general phase factor $\Delta\phi$. Include an expression for the relative length of the arms $\Delta L = L_2 - L_1$. Assume that the input field consists of parallel rays from a linearly polarized monochromatic source of wavelength λ (and $k = 2\pi/\lambda$) and amplitude E_0. Draw the setup. Where do the field maxima in the interference pattern occur ? [5]

(e) For a beam spitter with two input beams, show the relation:

$$\cos(\phi_2^r) + \cos(\phi_1^r) = 0,$$

is fulfilled in the general case that the relative phase shift between the reflection of the two input beams $\Delta\phi = \pi$, with ϕ_1^r and ϕ_2^r being the phases due to reflection of the two input beams on the beam splitter. Draw the two cosine functions. [4]

B2. Consider two dimensionless quadrature fields X_1 and X_2. In the photon number representation we can introduce the associated operators by using creation $\hat{a}^\dagger$ and annihilation $\hat{a}$ operators :

$$\hat{X}_1 = \frac{1}{2}(\hat{a}^\dagger + \hat{a}),$$

$$\hat{X}_2 = \frac{1}{2}i(\hat{a}^\dagger - \hat{a}).$$

(a) Evaluate the commutator $[\hat{X}_1, \hat{X}_2]$, using $[\hat{a}, \hat{a}^\dagger] = 1.$ [5]

(b) Find the uncertainty product $(\Delta X_1)^2(\Delta X_2)^2$. [Remark: The result should be in agreement with the uncertainty relation: $\Delta X_1 \Delta X_2 \geq 1/4$]. [2]

(c) The two field quadratures can be directly related to generalized position and momentum coordinates q and p, respectively, by:

$$\hat{X}_1 = (\frac{\omega}{2\hbar})^{1/2} q,$$

$$\hat{X}_2 = (\frac{1}{2\hbar\omega})^{1/2} p,$$

where ω is the angular frequency of a harmonic oscillator. Derive starting from $\Delta X_1 \Delta X_2 \geq 1/4$ the Heisenberg uncertainty relation for x and p_x. [You may use relations $q = \sqrt{m}x$ and $p = p_x/\sqrt{m}$.] [4]

(d) For the vacuum field the uncertainties for the two quadratures are identical:

$$\Delta X_1^{vac} = \Delta X_2^{vac} = 1/2.$$

Draw the phasor diagram for the vacuum state. [2]

(e) Calculate the magnitude of the vacuum field in a cavity volume of $1mm^3$ at 500nm. [2]

(f) Draw the phasor diagrams of quadrature squeezed states: the squeezed vacuum and the phase-squeezed light. Draw the experimental setup for detection of quadrature-squeezed vacuum states by the balanced homodyne technique. Why is the output phase sensitive? [5]

B3. A pulsed laser beam is focused to a spot of radius $1\mu m$ on a gas of atoms with a dipole moment of $\mu_{12} = 10^{-29} Cm$ at the laser frequency.

(a) Derive a formula for the pulse duration at FWHM. Calculate the pulse duration. Hint: Start by using a Gaussian field: $E_0(t) = E_{peak} \exp(-t^2/\tau^2)$, calculate the intensity at FWHM. [6]

(b) Use the formula for the pulse area to measure the rotation of the Bloch vector $\Theta = \frac{\mu_{12}}{\hbar} \int_{-\inf}^{+\inf} E_0(t) dt$ to calculate the peak energy of the laser pulse E_{peak}. [4]

(c) Now calculate the pulse energy required to rotate the Bloch vector by $\pi/2$ radians for Gaussian pulses with a duration (FWHM) of $1ps$. Hint: The pulse energy is the product of the area of the beam times the integral over the laser intensity. [5]

(d) If the system is initially in the ground state, find the state of the system at the end of the pulse. Draw the Bloch sphere. [5]

END OF PAPER