SEMESTER 2 EXAMINATION 2014-2015

MPHYS SYNOPTIC PHYSICS

Duration: 120 MINS (2 hours)

This paper contains 9 questions.

Answer **all** questions in **Section A**, **only one** question in **Section B**, and **only one** question in **Section C**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section C carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language word to word® translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

Section A

A1. Explain what is meant by dispersion in wave propagation and define the concepts of phase velocity and group velocity. [4]

A2. A diffraction grating is used to observe the sodium D lines of wavelengths $\lambda_1 = 589.0$ nm and $\lambda_2 = 589.6$ nm. If the grating has 300 rulings per millimeter, what is the angular separation of the two lines at first order? And at the highest order which can be observed? [4]

A3. The components of angular momentum obey the relation $[J_x, J_y] = i\hbar J_z$. Briefly illustrate the meaning of this relation. Why is it not possible, in general, to have states for which more than one component of $\boldsymbol{J}$ has a definite value, while it is possible to have states with simultaneously well-defined values of $\boldsymbol{J}^2$ (where $\boldsymbol{J}^2 = J_x^2 + J_y^2 + J_z^2$) and one component of $\boldsymbol{J}$? [4]

A4. Explain the difference between bosons and fermions. Why must there be two kinds of exchange symmetry? Show that in one of the two cases a doubly occupied state cannot exist (the Pauli exclusion principle). [4]

A5. Estimate the temperature (in degrees Kelvin) at which most of the atoms in a hydrogen gas in equilibrium will be ionised. [4]

Section B

B1. (a) Write down Maxwell's equations in vacuum in a region free of charges and currents. [4]

(b) By evaluating the curl of Faraday's law in differential form, obtain the wave equation for the electric field propagating through free space. What is the wave speed in this equation? [4]

(c) A monochromatic plane electromagnetic wave is incident along the normal to the plane interface between two dielectric media of electric permittivities ε_1 and ε_2.

(i) Write down the boundary conditions for the electric and magnetic fields at the interface between the two media. [4]

(ii) Applying the boundary conditions, show that the electric field amplitudes E_R and E_T for the reflected and transmitted waves as functions of the incident electric field amplitude E_I and of the permittivities ε_1 and ε_2 are given by

$$E_R = E_I \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} \quad , \qquad E_T = E_I \frac{2\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} \quad .$$

(Assume magnetic permeabilities $\mu_1 \simeq \mu_2 \simeq \mu_0$.) [4]

(iii) Determine the power transported per unit area by the incident, reflected and transmitted waves as a function of E_I, E_R and E_T. Verify that energy is conserved between the three waves. [4]

TURN OVER

B2. (a) Write down the time-independent and the time-dependent Schrödinger equations. [2]

Is it necessary for the wave function of a system to satisfy the time-dependent Schrödinger equation? Under what circumstances does the wave function of a system satisfy the time-independent Schrödinger equation? [2]

What is the significance of the Schrödinger equation being first-order in time, rather than second-order like Newton's equations of motion? [2]

(b) A particle is confined in a potential well such that its allowed energies are $E_n = n^2\varepsilon$, where $n = 1, 2, \ldots$ is an integer and ε is a positive constant. Let $u_1, u_2, \ldots, u_n, \ldots$ be the corresponding orthonormal energy eigenfunctions. At time $t = 0$ the particle is in the state described by the wave function

$$\psi(0) = 0.2u_1 + 0.3u_2 + 0.4u_3 + 0.843u_4 \quad .$$

(i) What is the probability, if the energy is measured at $t = 0$, of finding a value smaller than 6ε? [3]

(ii) What is the expectation value $\langle E \rangle$ of the energy of the particle at $t = 0$? [2]

(iii) What is its root mean square deviation ΔE? [2]

(c) Write down the wave function of the particle at time $t > 0$. [4]

(d) When the energy is measured it turns out to be 16ε. After the measurement, what is the wave function of the system? What result is obtained if the energy is measured again? [3]

Section C

C1. (a) The contribution from spin-orbit interaction to the Hamiltonian of hydrogen-like atoms can be written as

$$H_{\mathrm{s-o}} = -\frac{e}{m^2c^2}\frac{1}{r}\frac{\partial\phi}{\partial r}\,\boldsymbol{L}\cdot\boldsymbol{S}$$

where $\phi(r)$ is the Coulomb potential of the nuclear charge, r is the radial coordinate, $\boldsymbol{S}$ and $\boldsymbol{L}$ are the spin and orbital angular momentum. Briefly discuss the physical origin of this interaction. [4]

Give a valid set of quantum numbers to identify eigenstates of the atomic Hamiltonian once the spin-orbit interaction is included. [3]

(b) Using the above expression, or otherwise, show that the magnetic field experienced by the electron passing the nucleus is given by $\boldsymbol{B} \simeq Ze/(4\pi\varepsilon_0 mc^2 r^3)\boldsymbol{L}$. [4]

(c) Estimate B in the 2p configuration of hydrogen, using that the expectation value of r^{-3} in hydrogen-like states is $\langle r^{-3}\rangle = Z^3/[n^3a_0^3 l(l+1)(l+1/2)]$, where a_0 is the Bohr radius. [3]

From the above results, or otherwise, determine how the fine structure of hydrogen-like atoms scales with the atomic number Z. [3]

(d) As a result of relativistic fine-structure corrections, the energy level n splits into a multiplet of n distinct sublevels with energies

$$\Delta E_{nj} = -\frac{1}{2}\,mc^2\,\frac{(Z\alpha)^4}{n^3}\left(\frac{1}{j+1/2}-\frac{3}{4n}\right) .$$

What is the degeneracy of the highest sublevel of this multiplet? What is the degeneracy of the other sublevels? [3]

TURN OVER

C2. (a) Explain the concept of proper time τ. Given two events in space-time separated by a time-like interval $dx^\mu = (cdt, d\boldsymbol{x})$, illustrate the concept by evaluating the proper time $d\tau = \sqrt{dx^\mu dx_\mu}/c$ between the events. [4]

Show that

$$\frac{dt}{d\tau} = \gamma \, ,$$

where $\gamma = (1 - \boldsymbol{v}^2/c^2)^{-1/2}$. [2]

(b) Consider four-velocity

$$u^\mu = \frac{dx^\mu}{d\tau}$$

and four-momentum of a particle of mass m,

$$p^\mu = mu^\mu = (E/c \, , \; \boldsymbol{p}) \;\; ,$$

where E is the energy and $\boldsymbol{p}$ is the three-momentum.

Show that $E^2 = c^2\boldsymbol{p}^2 + m^2c^4$. [6]

(c) A charged pion at rest decays into a charged muon and a muon neutrino. Approximating the neutrino to be massless, determine the energy of the muon in terms of the pion mass and muon mass. [4]

(d) Two photons may collide to produce electron-positron pairs e^+e^-. High-energy photons of galactic origin pass through the cosmic microwave background radiation, which can be regarded as a gas of photons of energy 2.3 $\times 10^{-4}$ eV. What is the threshold energy of the galactic photons for the production of electron-positron pairs? [4]

END OF PAPER

