SEMESTER 1 EXAMINATION 2013/14

LASERS

Duration: 120 MINS

This paper contains 8 questions.

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

The emission cross-section of an optical transition , $\sigma(\omega)$, is given by the equation

$$\sigma(\omega) = \frac{\pi^2 c^2}{\omega_0^2 \tau_{rad}} g(\omega)$$

where ω_0 is the central frequency of the transition, τ_{rad} is its radiative lifetime, and $g(\omega)$ is a lineshape function with $\int g(\omega)\, d\omega = 1.$

The rate equations for the population inversion density, N_2, and the total photon number, ϕ, in a simple 4-level laser are:

$$\frac{dN_2}{dt} = \Gamma - \frac{(c/n)\sigma}{V}\phi N_2 - \frac{N_2}{\tau} \qquad \frac{d\phi}{dt} = V_a \frac{(c/n)\sigma}{V}\phi N_2 - \frac{\phi}{\tau_c}$$

where Γ is the pumping rate into the upper laser level, σ is the emission cross-section, n is the refractive index of the gain medium, V is the cavity mode volume, V_a is the volume of the mode in the gain medium, τ is the upper state lifetime, and τ_c is the cavity lifetime.

The electric field envelope, $A(x, y, z)$, of a Gaussian beam is given by

$$A(x, y, z) = \frac{A_0 w_0}{w(z)} \exp\left(-\frac{x^2 + y^2}{w(z)^2}\right) \exp\left(-\frac{ik(x^2 + y^2)}{2R(z)}\right) \exp\left(i \arctan\left(\frac{z}{z_0}\right)\right)$$

where A_0 is the field amplitude, w_0 is the radius of the beam waist at $z = 0$, w is the beam radius at position z, $k = \frac{2\pi}{\lambda}$, R is the radius of curvature of the beam wavefront at position z, and the Rayleigh length $z_0 = \frac{\pi w_0^2}{\lambda}$.

The spot size $w(z)$ of a Gaussian beam with its beam waist at $z = 0$ and propagating along the z-axis is given by

$$w(z)^2 = w_0^2 \left[1 + \frac{z^2}{z_0^2}\right].$$

Section A

A1. A laser cavity is formed by two mirrors with reflectivities R_1 and R_2, separated by length l. By considering the loss per round trip, or otherwise, show that the lifetime for photons in the cavity, τ_c, is given by

$$\tau_c = \frac{2l/c}{T_1 + T_2 - \log(L)}$$

where c is the speed of light, and $T_i = 1 - R_i$, and L is the total fractional loss in the cavity not associated with mirror transmission. Give two examples of processes which might contribute to the loss L.

(You may assume that $T_i \ll 1$, so that $\log(R_i) = \log(1 - T_i) = -T_i$.) [5]

A2. Describe physically, using diagrams if necessary, how a stack of pairs of layers of transparent materials whose refractive indices are different can act as a high reflector at a particular wavelength. Include in your answer any critical dimensions, such as the necessary layer thicknesses.

Give two reasons why such mirrors are usually used in preference to metallic mirrors in laser cavities. [5]

A3. The $^2F_{5/2} -^2 F_{7/2}$ transition in Yb^{3+} has a centre wavelength of 1020 nm, an excited state lifetime τ_{rad} = 1.5 ms, and a spectral width of 20 nm.

(a) Calculate the integrated optical cross-section, σ_0.

(b) Calculate the peak optical cross-section, σ_{peak}.

(c) How does the integrated optical cross-section compare to the physical

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cross-sectional area of the ion, if the ionic radius of the Yb^{3+} ion is 86 pm?

[5]

A4. Explain, using sketches of the respective gain spectra, why *homogeneously* broadened laser transitions generally only oscillate on a single cavity mode, but *inhomogeneously* broadened laser transitions can oscillate on many modes simultaneously. [5]

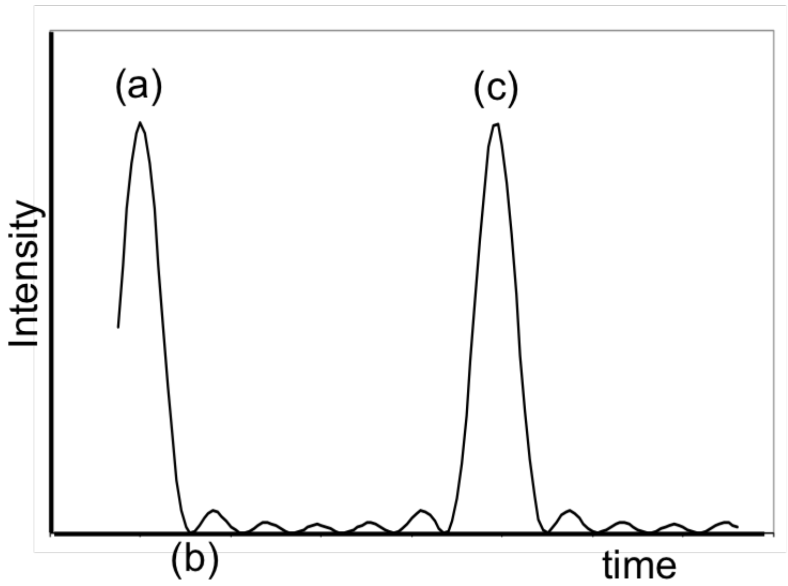
Section B

B1. (a) Describe using appropriate diagrams how *Kerr Lens Modelocking* can be used to produce short pulses from a Ti:sapphire laser. [4]

(b) The intensity variation with time of the mode locked pulse train from such a laser has the form

$$I \sim |E|^2 = E_0^2 \frac{\sin^2(N\Delta\omega\, t/2)}{\sin^2(\Delta\omega\, t/2)}$$

where E_0 is the electric field amplitude of each mode, N is the number of modes, and $\Delta\omega$ is the frequency spacing between modes. The variation is shown below.



By considering the time at points (a), (b), and (c), show that

(i) the time spacing between pulses is $2\pi/\Delta\omega$,

(ii) the approximate pulse width in time is $2\pi/N\Delta\omega$. [4]

(c) Describe physically what mode-locked operation corresponds to in terms

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of the photon distribution within the laser. [2]

(d) By writing down expressions for the peak power and the average power of a mode locked laser in terms of the pulse energy E, show that the ratio of the peak to the average power in such a mode locked laser is N. Hence for a Ti:sapphire laser with a gain curve extending from 750 nm to 850 nm, calculate the peak power for a laser of length 1.5 m which produces an average power of 0.4 W. [7]

(e) The peak intensity of each pulse is given by $I \sim N^2 E_0^2$. What happens to the pulse energy if the length of the cavity is doubled? [3]

B2. (a) The wave equation for the electric field of a monochromatic plane wave in free space can be written as

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2\right] E(x, y, z) = 0$$

where $k = \frac{2\pi}{\lambda}$. Show that if the electric field is written as $E(x, y, z) = A(x, y, z) \exp(ikz)$ then under some conditions the field envelope A satisfies the equation

$$\frac{\partial A^2}{\partial x^2} + \frac{\partial A^2}{\partial y^2} + 2ik\frac{\partial A}{\partial z} = 0.$$

What approximation must be made in deriving this expression? [4]

(b) Show that for a Gaussian beam with radius w transmitted through a circular aperture of radius a, the fractional power transmitted through the aperture is

$$P \sim 1 - \exp\left(-\frac{2a^2}{w^2}\right).$$ [3]

(c) Two coincident Gaussian beams with wavelengths λ_1 = 800 nm and λ_2 = 400 nm both propagate along the same axis, and both have a beam waist with radius w_0 = 10 μm at z = 0. Show that far from the beam waist, the ratio of the beam radii is given by

$$\frac{w_1}{w_2} = \frac{\lambda_1}{\lambda_2}.$$ [3]

(d) An aperture that is placed in the two beams at z = 1 cm transmits 99% of the power of the beam at 400 nm. What fraction of the power in the 800 nm beam is transmitted? [6]

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(e) How would the ratio of the transmitted powers at 400 nm and 800 nm through an aperture that transmitted 99% of the power at 400 nm compare to the ratio in part (d) if the distance to the aperture is changed to

(i) $z = 1$ m,

(ii) $z = 2\,\mu$m. [4]

B3. A laser is to be operated in *Q-switched* mode.

(a) (i) Show that the rate equation for N_2, the excited state population during the pumping phase of Q-switched operation is

$$\frac{dN_2}{dt} = \Gamma - \frac{N_2}{\tau}.$$

(ii) Show that the rate equations for N_2, the excited state population and ϕ, the total photon number during the emission phase of Q-switched operation are

$$\frac{dN_2}{dt} = -\frac{(c/n)\sigma}{V}\phi N_2$$

$$\frac{d\phi}{dt} = V_a\frac{(c/n)\sigma}{V}\phi N_2 - \frac{\phi}{\tau_c}$$

respectively, where the symbols are as defined in the rubric.

In both cases, describe what assumptions have been made in order to produce these equations. [6]

(b) Show that during the pumping phase, the time dependence of N_2 is

$$N_2 = \Gamma\tau\left[1 - \exp\left(-\frac{t}{\tau}\right)\right].$$ [3]

(c) Show that during the emission phase of operation, the peak photon number occurs when $N_2 = N_{\text{threshold}}$, where $N_{\text{threshold}}$ is the value of the population inversion when the laser is operating in continuous rather than Q-switched mode, which is given by

$$N_{\text{threshold}} = \frac{V}{V_a(c/n)\sigma\tau_c}.$$ [3]

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(d) Sketch in detail the photon number variation during pump phase and emission phase. [3]

(e) The overall efficiency of stored energy usage within the laser tends to 1 when the overpumping ratio, $x = \frac{N_2}{N_{th}}$, is greater than $\sim$5. What minimum pump rate is necessary to achieve $x = 5$ in a Nd^{3+}:YAG laser with a cavity length of 10 cm, and a gain medium of length 1 cm. The mirrors have reflectivity 1 and 0.95 respectively. The cross-section σ = 2.8 $\times$ 10^{19} cm^2, and the excited state lifetime τ = 230 μs. The refractive index of YAG is n =1.82. [5]

B4. (a) Using diagrams of the appropriate energy level structures, describe briefly the pumping mechanisms for

(i) HeNe lasers,

(ii) erbium-doped fibre lasers,

(iii) semiconductor lasers.

[6]

(b) For the three laser systems listed above, give a qualitative comparison of the typical operating efficiencies that you would expect. [3]

(c) An Er^{3+}-doped fibre amplifier (EDFA) at wavelength λ = 1.5μm has an Er^{3+} concentration of 10^{19} ions/cm^3, and length 10 m. Its core radius is 7μm. Calculate the stored energy in the fibre when the Er^{3+} ions are all inverted.

[4]

(d) The Er-doped fibre described in part (c) is used in a laser cavity with mirror reflectivities $R_1 = 1$, $R_2 = 0.99$. The output power obtained from the laser is 30 mW. How much energy is stored as photons within the cavity? (You may assume that the fibre refractive index n =1.5.) [3]

(e) How might suitable highly-reflecting mirrors be made within the core of the fibre itself? [4]

END OF PAPER