Phys6024 14-15 Answers

A1) (a)

- Population inversion is zero with no pumping
- Increases as pump increases [1]
- Reaches threshold value at the laser threshold, and does not increase any further as the pump increases. [2]

[Direct from lectures]

(b)

Limit of output power of cw laser depends on

- saturation intensity of the laser medium [2]
- area of the beam

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A2) During a single round trip, the photon number ϕ is changed:

$$\phi(t+\tau_{RT}) = \phi(t) \cdot R_1 R_2 \exp(-2\alpha L)$$

where R_1, R_2 are mirror reflectivities, and $\exp(-2\alpha L)$ is the absorption loss in the cavity.

This can be written as an exponential decay, so that

$$\phi(t+\tau_{RT}) = \phi(t) \exp\left(-\frac{\tau_{RT}}{\tau_c}\right) \qquad [1]$$

Equating these and taking log of both sides

$$\Rightarrow -\frac{\tau_{RT}}{\tau_c} = \ln R_1 + \ln R_2 - 2\alpha L \qquad [1]$$

If we assume that $R = 1-T$ and $T \ll 1$, then $\ln R = \ln(1-T) \sim -T$ [1]

Thus $$\tau_c = \frac{\tau_{RT}}{T_1 + T_2 + 2\alpha L} = \frac{2L/c}{T_1 + T_2 + 2\alpha L}$$

[standard proof from lectures]

(a) $L = 20\text{cm}$, $T = 0.005$, $\alpha L = 0 \Rightarrow \tau_c = 53\,\mu\text{s}$ [1]

(b) $L = 2\text{cm}$, $T_1 = 0$, $T_2 = 0.1$, $\alpha L = 0 \Rightarrow \tau_c = 13.3\,\text{ns}$ [1]

[simple calculations]

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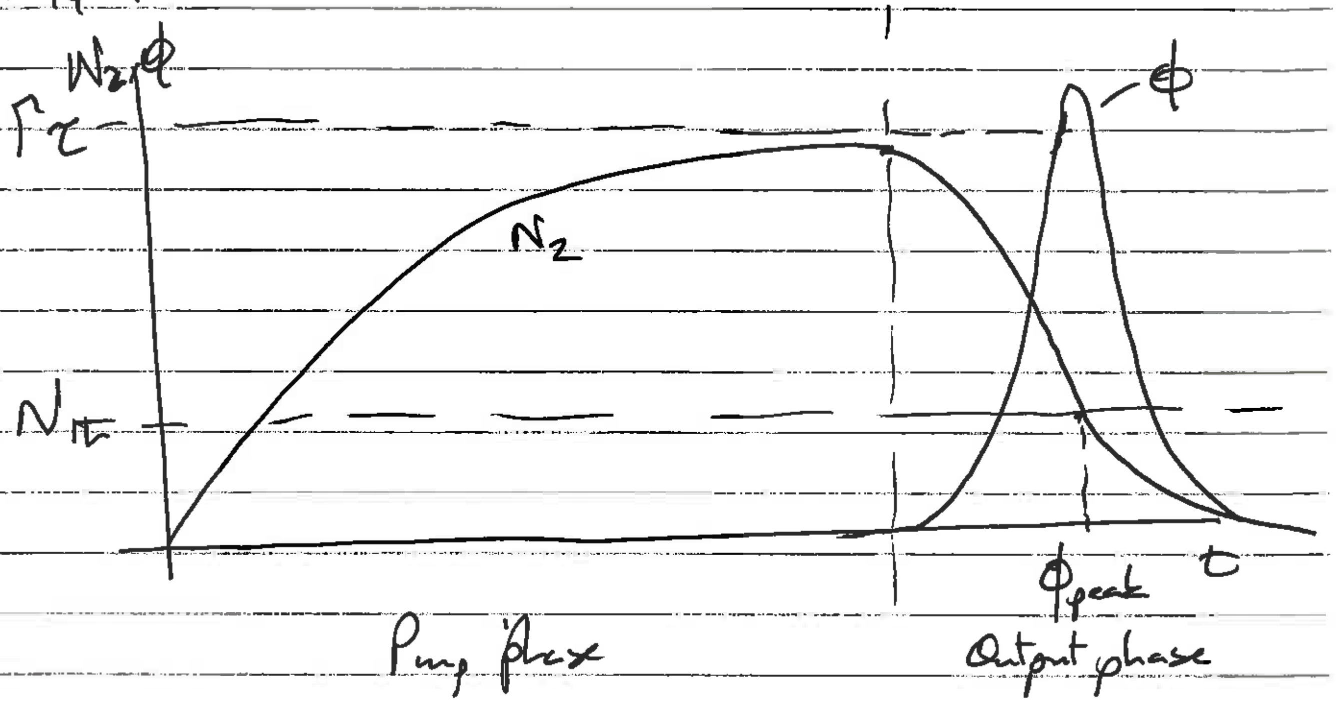
A3



Marks for: Pump phase – form of graph
axes labels
N_2 tending to $\Gamma\tau$ [2]

Output phase: form of graphs [2]
ϕ_{peak} @ $N_2 = N_{threshold}$ [1]

[Synthesis of 2 areas of lecture notes]

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A4) a) Wavefront is flat at beam waist [1]

b) For $\lambda = 800\,nm$, $\omega_L = 5\,mm$, $f = 2\,cm$

$$\Rightarrow \omega_0 = \frac{\lambda f}{\pi \omega_L} = 1.02 \times 10^{-6}\,m$$

$$= 1\,\mu m.$$ [1]

[sample calculation]

c) $P = 10\,W.$

$$I = I_0 e^{-2r^2/\omega^2}, \quad P = \int I \, dx\,dy$$

$$\Rightarrow P = \iint I_0 e^{-2r^2/\omega^2} dx\,dy = \iint I_0 e^{-2r^2/\omega^2} r\,dr\,d\theta$$

$$= 2\pi I_0 \int r e^{-2r^2/\omega^2} dr$$

$$= 2\pi I_0 \left[-\frac{\omega^2}{4} e^{-2r^2/\omega^2} \right]_0^{\infty} = I_0 \frac{\pi \omega^2}{2}$$ [2]

$$\Rightarrow I_0 = \frac{2P}{\pi \omega^2} = \underline{\underline{6.12 \times 10^{12}\,W m^{-2}}}$$ [1]

[similar calculation done in notes]

(1st is acceptable in calculation)

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B1 a) $$A(z,t) = c \cdot \exp -\left(\frac{t^2}{2k''(z_c + iz)}\right) \quad \text{where } z_c = \frac{\tau_0^2}{2k''}$$

$$= c \exp -\left[\frac{t^2}{2k''(z_c+iz)}\left(\frac{z_c - iz}{z_c - iz}\right)\right]$$

$$= c \exp -\left[\frac{t^2}{2k''} \cdot \frac{z_c - iz}{z_c^2 + z^2}\right]$$

$$= c \exp -\left[\frac{t^2 z_c}{2k''(z_c^2+z^2)} - \frac{i z t^2}{2k''(z_c^2+z^2)}\right] \quad [2]$$

Taking the real part, which represents the amplitude of the pulse. [1]

$$\frac{t^2 z_c}{2k''(z_c^2+z^2)} = \frac{t^2}{\frac{\tau_0^2}{z_c}(z_c^2+z^2)} \Rightarrow \frac{t^2}{\tau_0^2\left(1+\frac{z^2}{z_c^2}\right)} \quad [2]$$

so the amplitude of the pulse $= \exp -\frac{t^2}{\tau^2}$, where $\tau^2 = \tau_0^2\left(1+\frac{z^2}{z_c^2}\right)$

b) Imaginary part of $A(z,t)$ is $\sim \exp\left(\frac{-izt^2}{2k''(z_c^2+z^2)}\right)$

~~$\sim \exp$~~

Can write overall field envelope as

$$A(z,t) \sim \exp(-at^2 + ibt^2)$$

The field E is given by $E(z,t) \sim \exp(-at^2 + ibt^2)\exp(-i\omega_0 t)$

B1 (cont)

$$\Rightarrow \quad E(z,t) \sim \exp(-at^2)\exp(-i(\omega_0 t - bt^2))$$

Gaussian envelope, with phase given by $\phi = \omega_0 t - bt^2$ [2]

The instantaneous frequency is = rate of change of phase [1]

$$\Rightarrow \omega_{inst}(t) = \frac{d\phi}{dt} = \omega_0 - 2bt \qquad [1]$$

= Linear frequency shift with time

c) If pulse lengthens by 10%, then $\tau = 1.1\,\tau_0$

From part (a), $\tau^2 = \tau_0^2\left(1 + \frac{z^2}{z_c^2}\right)$, $z_c = \frac{\tau_0^2}{2k''}$

Thus here, $\tau^2 = 1.21\,\tau_0^2 \quad \Rightarrow 1 + \frac{z^2}{z_c^2} = 1.21$

$$\Rightarrow \frac{z^2}{z_c^2} = 0.21 \qquad [2]$$

z_c is given by $\frac{\tau_0^2}{2k''} \quad \Rightarrow \quad \tau_0^2 = \frac{2k''z}{\sqrt{0.21}}$

$$= \frac{2 \cdot 350 \cdot 4}{\sqrt{0.21}} = 6.11 \times 10^3 \text{ fs}^2$$

$$\Rightarrow \tau_0 = 78\text{ fs}. \qquad [1]$$

B1 (cont)

d) Equation is $\tau^2 = \tau_0^2\left(1 + \frac{z^2}{z_c^2}\right)$, but $z_c = \frac{\tau_0^2}{2k''}$

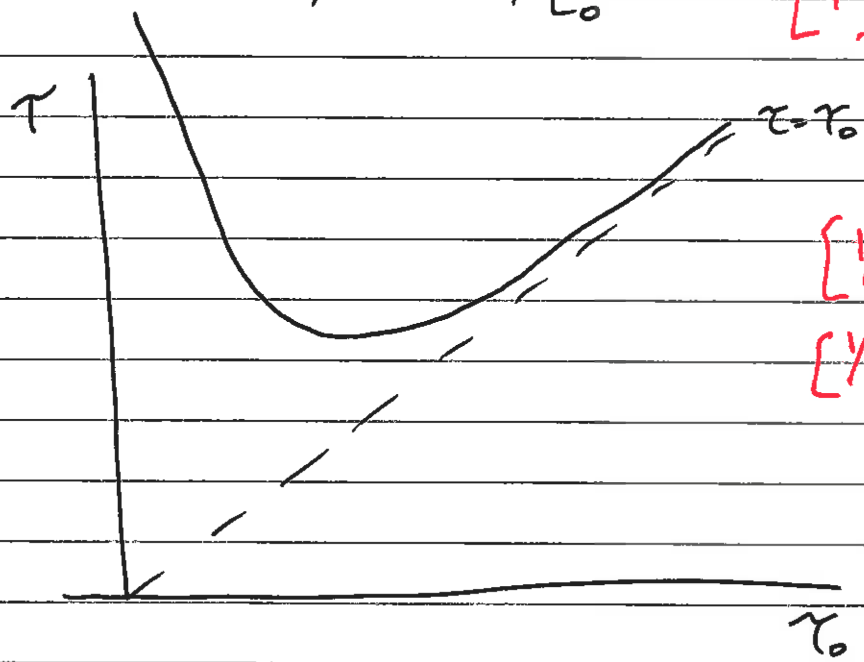
So full equation is

$$\tau^2 = \tau_0^2 + \frac{4k''^2 z^2}{\tau_0^2}$$

Hence, at large τ_0, $\tau \sim \tau_0$ [1]

at small τ_0, $\tau \sim 1/\tau_0$ [1]

Sketch:



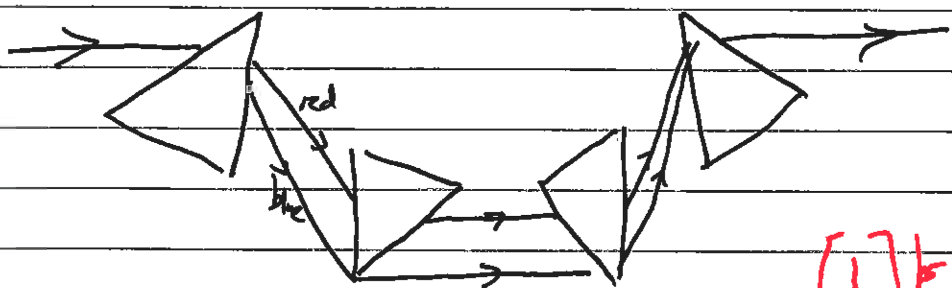
[½] for form

[½] for correct labelling of axes + lines

e) Optical systems include:

1) Prism (or grating) compressor

Diagram:



[1] for form

[1] for precise layout, labelling

B1 (cont)

Explanation

- light propagating into first prism is dispersed spectrally,
- prism (2) creates spatially dispersed beam
- prism (3) converges spectral components
- prism (4) recombines all spectral components into a single beam.

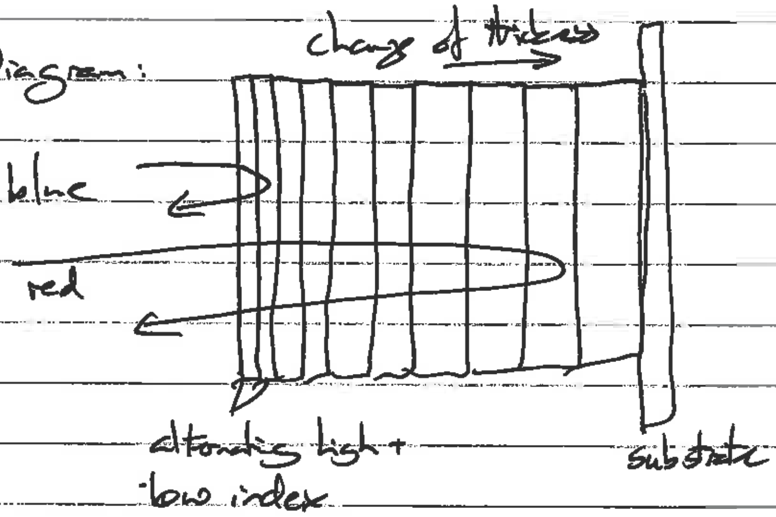
[2]

Total path length for red light is longer than for blue, because of the ~~amp~~ extra path through glass in prisms (2) + (3).

Thus red light is delayed with respect to blue, and dispersion is opposite to that produced propagating through glass.

[1]

Or 2) Chirped mirrors: Diagram:



[2]

- Pairs of high + low index layers of thickness $\lambda/4$ each create high reflectivity

[1]

- depth at which light is reflected depends on the

B1 (cont)

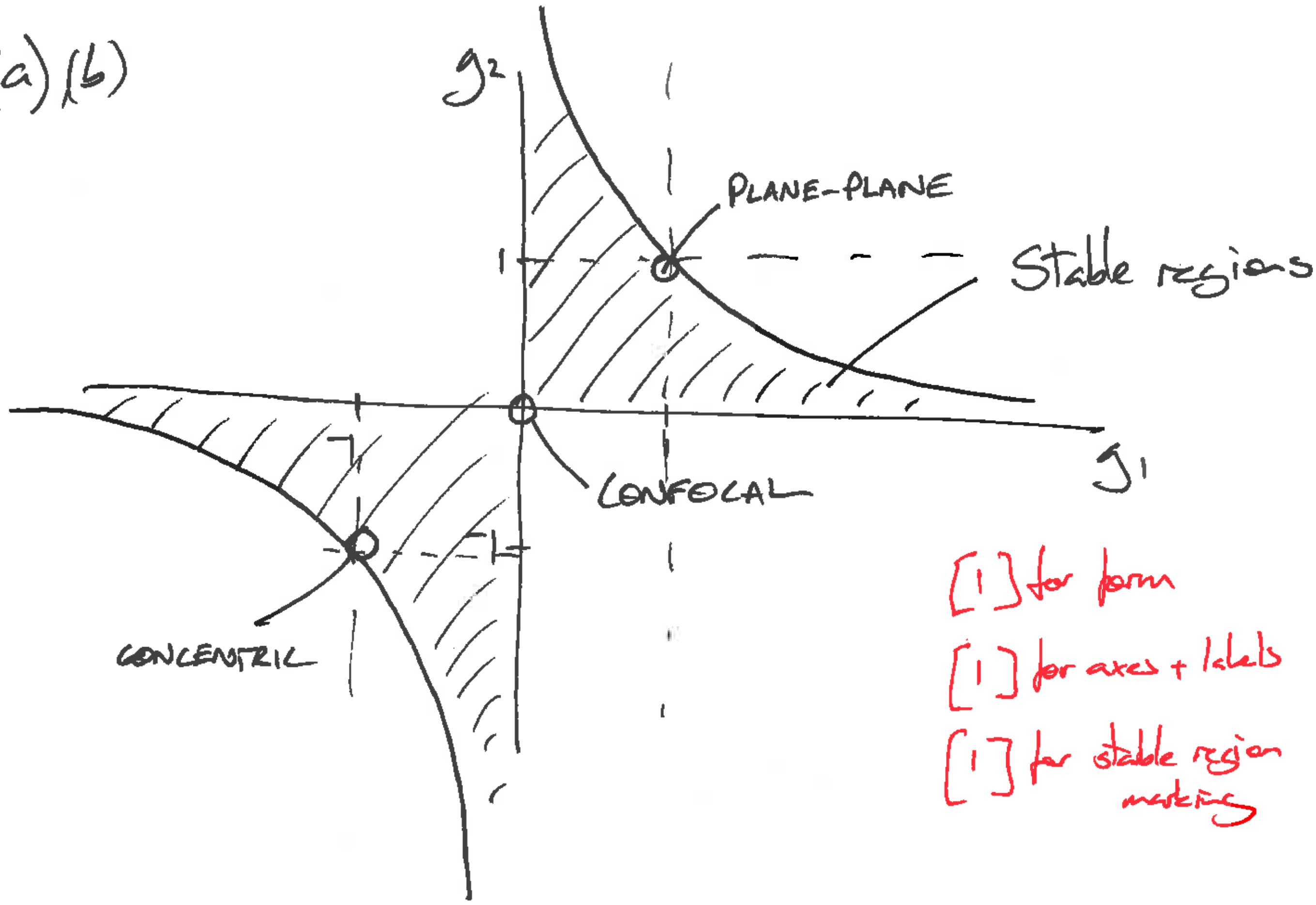
change in layer thickness with depth

- blue light reflected near surface
- red light reflected near substrate

Thus red light is delayed with respect to blue, and dispersion is opposite to ~~that~~ produced by glass.

[2]

B2) (a) (b)



[1] for form
[1] for axes + labels
[1] for stable region marking

b) i) Confocal, $L=R \Rightarrow g_1, g_2 = 0$ [1]

ii) concentric: $L=2R \Rightarrow g_1, g_2 = 1-\frac{2R}{R} = -1$ [1]

iii) Plane, plane: $R=\infty \Rightarrow g_1, g_2 = 1$ [1]

c) $$z_0^2 = \frac{g_1 g_2 (1-g_1 g_2)}{(g_1+g_2 - 2g_1 g_2)^2} L^2$$

here, cavity is symmetric so $g_1 = g_2$

$$\Rightarrow z_0^2 = \frac{g^2(1-g^2)}{(2g-2g^2)^2} L^2$$

$$z_0^2 = \frac{g^2(1-g^2)L^2}{4g^2(1-g)^2} = \frac{L^2}{4}\frac{(1-g)(1+g)}{(1-g)^2}$$

$$= \frac{L^2}{4}\frac{(1+g)}{(1-g)}$$ [2]

for confocal, $g=0 \Rightarrow z_0^2 = \frac{L^2}{4} \Rightarrow \underline{\underline{z_0 = \frac{L}{2}}}$

So: $z_0 = \frac{\pi w_0^2}{\lambda} = \frac{L}{2} \Rightarrow w_0^2 = \frac{\lambda L}{2\pi}$

$$\underline{\underline{w_0 = \sqrt{\frac{\lambda L}{2\pi}}}}$$ [2]

d) For free-space Gaussian, $w(z)^2 = w_0^2\left(1 + \frac{z^2}{z_0^2}\right)$

$\Rightarrow$ Here, $z = \frac{L}{2}$ (as beam waist is in centre of cavity) [1]

$z_0 = \frac{L}{2}$

$\Rightarrow w_{end}^2 = w_0^2 \cdot (1+1) \Rightarrow \underline{\underline{w_{end} = \sqrt{2}\, w_0}}$ [1]

for 99% transmission, $T = 1 - \exp\left(\frac{-2a^2}{w^2}\right) = 0.99$

$\Rightarrow \exp\left(\frac{-2a^2}{w^2}\right) = 0.01$, $\frac{-2a^2}{w^2} = \log(0.01) = -4.6$

$$\Rightarrow a^2 = \frac{4.6\, w^2}{2}$$

$$\Rightarrow \underline{\underline{a = 1.5\, w}}$$ [2]

here, $\omega_{end} = \sqrt{2} \sqrt{\frac{\lambda L}{\pi}} = 201\,\mu m$

$\Rightarrow$ for $T = 0.99$, $a = 1.5\omega$

$\Rightarrow$ diameter $= 3\omega$

$\sim 600\,\mu m$ [1]

e) TEM_{11} is physically larger than TEM_{00}, so loss when reflected off a $600\,\mu m$ mirror will be higher [1]

f) High reflectivity can be created using BRAGG MIRROR

- Consists of N pairs of layers of high + low index material, each with optical thickness $\frac{1}{4}\lambda$ [1]
- Reflections from surface of each pair are in phase [1], as one has a π phase shift from interface reflection and the other has a π phase shift from propagation [1]
- High reflectivity when N is large [1]

B3)

a) Γ: – pumping rate Eqn ①

$\frac{c/n\,\sigma}{V}\phi N_2$ – rate of stimulated emission per unit vol.

$\frac{N_2}{\tau_2}$ – rate of spontaneous emission

[2]

Eqn ②

$V_a \frac{c/n\,\sigma}{V}\phi N_2$ – total stimulated emission rate

$-\frac{\phi}{\tau_c}$ – rate of loss of photons from cavity

[2]

b) from equation ②, at steady state $\frac{d\phi}{dt} = 0$

$$\Rightarrow \frac{V_a}{V}(c/n)\sigma\,\phi N_2 = \frac{\phi}{\tau_c}$$

So for $\phi \neq 0$, $N_2 = \frac{V}{V_a (c/n)\sigma\,\tau_c}$ which is independent of ϕ

[2]

c) From eqn for N_2:

$$\frac{c/n\,\sigma}{V}\phi N_2 = \Gamma - \frac{N_2}{\tau} \quad \text{as } \frac{dN}{dt}=0$$

At steady state, $N_2 = \dfrac{V}{V_a\, c/n\, \sigma\, \tau_c}$ [1]

$$\Rightarrow \phi = \left(\frac{V}{c/n\,\sigma}\right)\left[\frac{\Gamma}{N_2} - \frac{1}{\tau}\right]$$

$$= \frac{V}{c/n\,\sigma}\left[\frac{\Gamma V_a\, c/n\,\sigma\,\tau_c}{V} - \frac{1}{\tau}\right]$$

$$\Rightarrow \phi = \Gamma V_a \tau_c - \frac{V}{c/n\,\sigma\,\tau} \quad \text{as required.}$$ [2]

d) Outpower $\sim \phi T$

To find maximum output power, take $\dfrac{dP}{dT} = 0$. [1]

$$\phi T = \left[\Gamma V_a \tau_c - \frac{V}{c/n\,\sigma\,\tau}\right] T$$

but $\tau_c = \dfrac{2L}{c/n(T+U)}$

$$\Rightarrow \phi T = \frac{\Gamma V_a\, 2L\, T}{c/n(T+U)} - \frac{VT}{c/n\,\sigma\,\tau}$$ [1]

$$\frac{d}{dT}(\phi T) = \frac{\Gamma V_a\, 2L}{c/n}\left[\frac{1}{(T+U)} - \frac{T}{(T+U)^2}\right] - \frac{V}{c/n\,\sigma\,\tau}$$

$$\frac{d}{dT}(\Phi T) = \frac{\Gamma V_a 2L}{c/n}\left[\frac{U}{(T+U)^2}\right] - \frac{V}{c/n\,\sigma\tau} = 0 \text{ for max/min}$$ [1]

$$\Rightarrow \frac{\Gamma V_a 2L}{c/n}\,\frac{U}{(T+U)^2} = \frac{V}{c/n\,\sigma\tau}$$

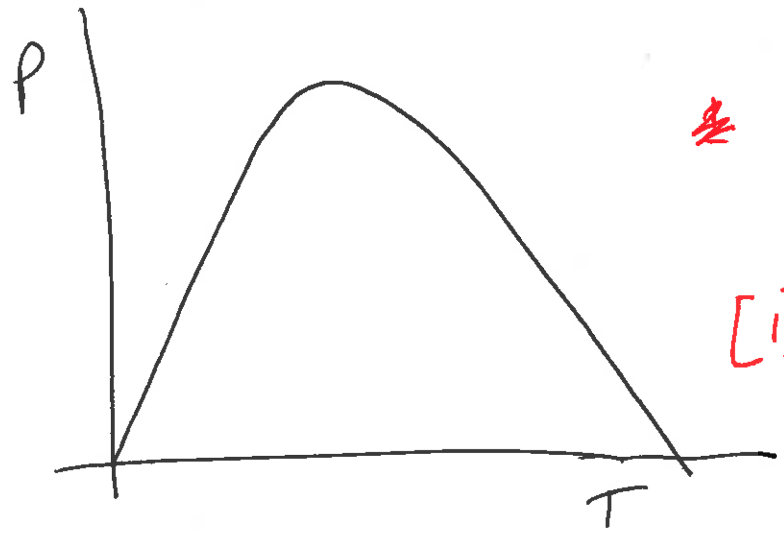
Rearrange to find T:

$$(T+U)^2 = \frac{c/n\,\sigma\tau}{V}\cdot\frac{\Gamma V_a 2L U}{c/n}$$

$$\Rightarrow T+U = \sqrt{\Gamma \frac{V_a}{V}\sigma\tau 2LU}$$

$$\Rightarrow T = \sqrt{\Gamma \frac{V_a}{V}\sigma\tau 2LU} - U$$ as required. [2]

e)



[1]

Physically – for small $T \to 0$, output power goes to zero as no transmission of light out of cavity [1]

– for T large, laser threshold gets too high for ~~cont~~ oscillation to occur. [1]

B3 (cont)

f) sources of useless loss could include

- diffraction around mirrors
- absorption by impurities in gain medium
- scattering within gain medium ~~or cavity~~
- absorption in mirrors

[1] each, up to [3]

B4)

a) Schawlow–Townes limit arises from spontaneous emission into laser mode [1]

- phase of spontaneous emission is random [1]
- adds to stimulated emission and causes drift in overall output phase + amplitude
- phase uncertainty results in increase in frequency width of laser output [2]

b) $\lambda = 1.06\,\mu m$, $P = 10W$.

First calculate $\tau_c = \dfrac{2L/c}{T_1 + T_2 + 2\alpha L} \approx \dfrac{2L}{cT}$ where $T = 0.1$ [1]

$\Rightarrow \tau_c = 20\,ns$

$$\Rightarrow \Delta\omega_{ST} = \frac{\hbar\omega}{P} \cdot \frac{1}{2\tau_c^2} = \frac{hc}{\lambda P} \cdot \frac{1}{2\tau_c^2} = 2.34\times10^{-5}\ \text{rad s}^{-1}$$

as required. [2]

c) Mode frequency for q^{th} mode $= \nu_q = \dfrac{c}{\lambda_q} = q \cdot \dfrac{c}{2L}$

Thus $\Delta\nu_q = q \cdot -\dfrac{c}{2L^2}\Delta L = \nu_q \dfrac{\Delta L}{L}$

$$\Rightarrow \frac{\Delta\nu}{\nu} = \frac{\Delta L}{L}$$ [2]

Here $\Delta\nu = \dfrac{2.34\times10^{-5}}{2\pi} = 3.7\times10^{-6}$, $\nu = \dfrac{c}{\lambda}$

$$\Rightarrow \frac{\Delta\nu}{c}\lambda = \frac{\Delta L}{L} = 1.32\times10^{-20}$$ [1]

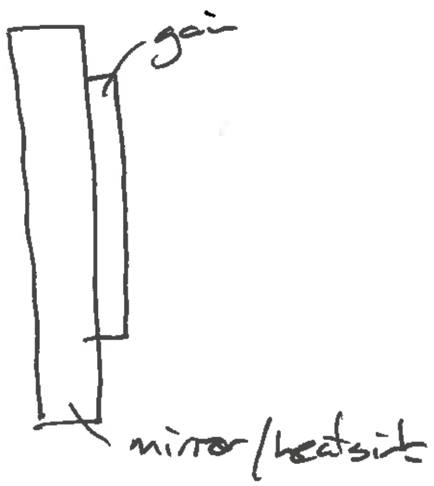
c) (cont)

Thus $\alpha \Delta T = \frac{\Delta L}{L} = \Delta\nu \cdot \frac{\lambda}{c}$ [1]

$\Rightarrow \Delta T = \Delta\nu \cdot \frac{\lambda}{c} \cdot \frac{1}{\alpha} = 1.6 \times 10^{-15}$ K. [1]

d) Disk lasers:

In a disk laser, gain medium is very thin [1], and bonded to a heat sink which acts as a mirror [1]



Heat generated in disk is conducted away into the heat sink,
$\Rightarrow$ Thermal gradient is axial, not radial, so no thermal lensing occurs. [1]

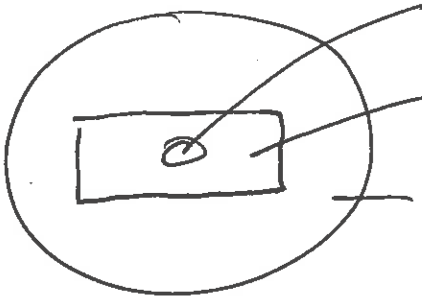
Thin gain medium means pump absorption is <u>low</u>, so pump must be <u>recycled</u> – reflected repeatedly back onto the gain medium. [1]

B4 (3)

e) Cladding pumping

- Mode quality of high power diodes is poor, so coupling into small fibre cores is hard [1]

- Cladding pumping ~~[illegible]~~ uses a fibre with an inner core, which is doped, surrounded by an outer core which is large + undoped.



[1 2]

Pump light can be launched efficiently into the outer core, and is absorbed into the inner core, which carries the laser light. [1]

- Fibres have a very high surface to volume ratio, so heat generated in the active region can dissipate rapidly, avoiding thermal lensing.
The guided mode nature of the fibre also reduces the effect of thermal distortions. [1]