SEMESTER 1 EXAMINATION 2014-2015

LASERS

Duration: 120 MINS (2 hours)

This paper contains 8 questions.

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

In a stable two mirror resonator, the Rayleigh length, z_0, of the cavity mode is given by

$$z_0^2 = \frac{g_1 g_2 (1 - g_1 g_2)}{(g_1 + g_2 - 2 g_1 g_2)^2} l^2 \ ,$$

where g_i are given by

$$g_i = 1 - \frac{l}{R_i} \ ,$$

where R_i is the radius of curvature of the mirror, and l is the length of the cavity

For a Gaussian beam propagating in free space,

$$w^2 = w_0^2 \left(1 + \frac{z^2}{z_0^2} \right) \ ,$$

where w is the spot size, z is the distance propagated fro the beam waist, and z_0 is the Rayleigh length, given by $z_0 = \pi w_0^2 / \lambda$, and w_0 is the spot size at the beam waist.

The fractional transmission of a Gaussian beam with spot size w through an aperture of radius a is given by:

$$T = 1 - \exp(-2a^2/w^2) \ .$$

Section A

A1. (a) Describe how the population inversion inside a continuous wave (cw) laser varies as the pumping level is increased from zero to above the laser threshold. [3]

(b) Give two physical parameters of a cw laser system which might limit the maximum output power available from the laser. [2]

A2. Show that for a simple optical cavity comprising two mirrors with reflectivities R_1 and R_2, separated by distance l, and a round trip loss given by $\exp(-2\alpha l)$, the lifetime of a photon within the cavity is given by

$$\tau_c = \frac{2l/c}{T_1 + T_2 + 2\alpha l} ,$$

where $T_i = 1 - R_i$, and you may assume that $T_i << 1$.

Calculate the cavity lifetime of:

(a) a HeNe laser with length 20 cm, and $R_1 = R_2 = 0.995$.

(b) a Nd:YAG with length 2 cm and $R_1 = 1, R_2 = 0.9$. [5]

A3. Draw a sketch of the variation of both population inversion and photon number with time during

(a) the pumping phase

(b) the output phase

of Q-switched laser operation. [5]

TURN OVER

A4. (a) What shape is the wavefront of a Gaussian beam at a beam waist?

(b) The focal spot formed by a lens with focal length f is given approximately by:

$$w_0 = \frac{\lambda f}{\pi w_L} \, ,$$

where λ is the wavelength of the light, and w_L is the spot size (i.e. the beam radius at $1/e^2$ of its maximum intensity) of the Gaussian beam at the lens. Calculate the spot size you would expect for a beam with wavelength $\lambda = 800\,\mathrm{nm}$ and $w_L = 5\,\mathrm{mm}$ incident on a lens with focal length $f = 2\,\mathrm{cm}$.

(c) If the power of the beam is $10\,\mathrm{W}$, what is the peak intensity at the focus? You may assume that the intensity distribution is

$$I = I_0 \exp\left(-\frac{2r^2}{w^2}\right) .$$

[5]

Section B

B1. The envelope of a Gaussian pulse propagating among the z axis in a dispersive medium can be shown to be

$$A(z,t) = \frac{A_0}{\sqrt{z_c + iz}} \exp\left[-\frac{t^2}{2k''(z_c + iz)}\right] ,$$

where A_0 is a constant, $k'' = \frac{d^2k}{dt^2}$ and $k = 2\pi n/\lambda$, where n is the refractive index of the medium. The constant z_c is given by:

$$z_c = \frac{\tau_0^2}{2k''} ,$$

where τ_0 is the minimum pulse length.

(a) Show that the pulse length at distance z, $\tau(z)$, varies as:

$$\tau(z)^2 = \tau_0^2 \left[1 + \frac{z^2}{z_c^2}\right] .$$ [5]

(b) Show that the form of the imaginary part of $A(z,t)$ produces a linear frequency shift in time through the pulse after propagating through a dispersive medium. [4]

(c) A pulse centred at $\lambda = 800\,\mathrm{nm}$ propagates through an optical system made of a dispersive glass with $k'' = 350\,\mathrm{fs}^2\,\mathrm{cm}^{-1}$. The total length of glass through which the pulse passes is $4\,\mathrm{cm}$. How short must the pulse be for the dispersion of the glass to have a significant effect on pulse length (in this case, a significant effect might mean lengthening by 10 %). [3]

(d) By considering the behaviour of τ at very large and very small values of τ_0, sketch a graph of the variation of the output pulse τ from a dispersive system like that described above as the input pulse length τ_0 varies. [3]

(e) Describe, with the aid of a diagram, an optical system which could be used to reverse the effects of the chirp produced by propagation through glass. [5]

TURN OVER

B2. (a) In a two mirror cavity, the mirror g parameters are given by

$$g_i = 1 - \frac{l}{R_i} ,$$

where l is the length of the cavity, and R_i is the radius of curvature of the mirror. Show on a graph of g_2 vs. g_1 the regions in which the cavity is stable. [3]

(b) Mark on your diagram the positions corresponding to

(i) confocal
(ii) concentric
(iii) plane-plane

cavities. [3]

(c) A confocal resonator is to be used as a cavity for a HeNe laser, at a wavelength of $632.8\,\mathrm{nm}$. The cavity length $l = 20\,\mathrm{cm}$. Show that the Rayleigh length, z_0, of the beam inside the cavity is given by

$$z_0 = \frac{l}{2} ,$$

and hence show that the spot size at the centre of the cavity is

$$\omega_0 = \sqrt{\frac{\lambda l}{2\pi}} .$$ [4]

(d) By considering beam propagation from the centre to the end of the cavity, show that the mirrors should have a diameter of at least $\sim 600\,\mu\mathrm{m}$ in order that diffraction loss at the mirrors is less than 1%. [5]

(e) Would you expect the loss for a higher order cavity mode, such as the TEM_{11} mode, to be high or lower than the loss for the TEM_{00} (Gaussian beam) mode described above? Give a brief reason for your answer. [1]

(f) Mirrors for a HeNe laser need to have high reflectivity because of its low gain. Describe how such high-reflectivity mirrors can be constructed and explain briefly how they operate. [4]

B3. The rate equations for the population inversion density, N_2, and the total photon number, ϕ, in a simple 4-level laser are:

$$\frac{dN_2}{dt} = \Gamma - \frac{(c/n)\sigma}{V}\phi N_2 - \frac{N_2}{\tau} \quad \text{and} \quad \frac{d\phi}{dt} = V_a\frac{(c/n)\sigma}{V}\phi N_2 - \frac{\phi}{\tau_c} \, ,$$

where Γ is the pumping rate into the upper laser level, σ is the emission cross-section, n is the refractive index of the gain medium, V is the cavity mode volume, V_a is the volume of the mode in the gain medium, τ is the upper state lifetime, and τ_c is the cavity lifetime.

(a) Explain physically what each term in these two equations describes. [4]

(b) Show that when the laser is operating in the steady state, the population inversion, N_2, is independent of the output power. [2]

(c) Show that when the laser is operating in the steady state, the photon number ϕ is given by

$$\phi = \Gamma V_a \tau_c - \frac{V}{(c/n)\sigma\tau} \, .$$ [3]

(d) Given that the laser output power, P, is proportional to ϕT, show that there exists an optimum value for T given by

$$T = \sqrt{\frac{V_a}{V} 2l\Gamma\sigma\tau U} - U \, ,$$

where the laser output power will be maximised. [5]

(The cavity lifetime, τ_c can be written as

$$\tau_c = \frac{2l}{(c/n)(T+U)} \, ,$$

where T is the transmission of the output coupler and U represents the *useless* loss in the cavity, for example absorption loss.)

(e) Sketch a graph of the variation of P with T, and describe physically why the graph has this form. [3]

(f) Give three sources of *useless* loss found in laser cavities. [3]

TURN OVER

B4. The Schawlow-Townes limit describes the fundamental narrowest frequency width which can be observed for a cw laser. The narrowest frequency width $\Delta\omega_{ST}$ is given by

$$\Delta\omega_{ST} = \frac{\hbar\omega}{P}\frac{1}{2\tau_c^2},$$

where P is the laser power, ω is the angular frequency of the laser, and τ_c is the cavity lifetime.

(a) Explain qualitatively what determines this fundamental limit on the linewidth of a narrow-band laser. [4]

(b) For a Nd:YAG laser with wavelength $1.06\,\mu\mathrm{m}$ and output power of $10\,\mathrm{W}$, show that the Shawlow-Townes limit $\Delta\omega_{ST} = 2.3 \times 10^{-5}\mathrm{rad\,s^{-1}}$ if its cavity is $30\,\mathrm{cm}$ long and the mirrors have reflectivity $R_1 = 1$ and $R_2 = 0.9$. [3]

(c) The cavity comprises a Nd:YAG crystal with mirrors bonded to each end. What change in temperature of the crystal produces a frequency shift in the laser modes equal to the Schawlow-Townes width? [5]

(The thermal expansion of a solid is given by $\frac{\Delta l}{l} = \alpha\Delta T$, where α is the thermal expansion coefficient, which for Nd:YAG is $8 \times 10^{-6}K^{-1}$)

(d) In order to reach the narrow linewidths necessary for high precision interferometry experiments, very high average power lasers must be used. Describe how the disk laser geometry allows very high power cw operation. Include in your answer an explanation of how problems of thermal lensing are avoided and how the lasers are pumped. [4]

(e) Describe how the process of *cladding pumping* allows the production of very high power diode-pumped fibre lasers. What other features of fibre lasers are important in allowing high average powers to be produced? [4]

END OF PAPER