SEMESTER 1 EXAMINATION 2012/13

# LASERS

Duration: 120 MINS

*Answer **all** questions in **Section A** and two **and only two** questions in **Section B**.*

*Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.*

*A Sheet of Physical Constants will be provided with this examination paper.*

*An outline marking scheme is shown in brackets to the right of each question.*

*Only university approved calculators may be used.*

For a Gaussian beam propagating in free space with a Rayleigh length $z_0 = \pi w_0^2/\lambda$

$$w^2(z) = w_0^2 \left[1 + \frac{z^2}{z_0^2}\right]$$

where $z$ is the distance propagated from the beam waist, $\lambda$ is the wavelength of the light, $w$ is the spot size (radius at which the beam intensity is $1/e^2$ of its maximum value), and $w_0$ is the minimum spot size (the beam waist size).

The reflectivity $R$ of a stack of $N$ quarter-wave layers of refractive index $n_1$ and $n_2$ on a substrate with index $n_T$ is given by

$$R = \left[\frac{1 - \frac{n_T}{n_0}\left(\frac{n_2}{n_1}\right)^{2N}}{1 + \frac{n_T}{n_0}\left(\frac{n_2}{n_1}\right)^{2N}}\right]^2$$

where $n_0$ is the index of the surrounding medium.

The ABCD matrices for various optical elements are:

Drift of distance $d$: $\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$

Reflection at a mirror with radius of curvature $R$: $\begin{pmatrix} 1 & 0 \\ -\frac{2}{R} & 1 \end{pmatrix}$

Refraction at a lens of focal length $f$: $\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$

and all ABCD matrices beginning and ending within a medium of the same refractive index have $AD - BC = 1$.

# Section A

**A1.** A laser cavity with length $l$ is formed of two mirrors with radii of curvature $R_i$. What condition is required of the laser beam wavefronts within the cavity to ensure a stable cavity mode?

For each mirror, the parameter $g_i$ is defined by $g_i = 1 - l/R_i$. On a diagram of $g_1$ vs. $g_2$, sketch the region where the cavity will be stable. [5]

**A2.** Define what is meant by the *threshold* of a continuous wave laser.

Describe the process of *gain clamping* in a continuous wave laser.

How will the threshold of a laser change as the output coupling is increased? [5]

**A3.** Sketch on a single diagram the variation of (a) intensity and (b) wavefront radius of curvature of a Gaussian beam as it passes through a focus.

Show that the product of the waist size $w_0$ and the beam divergence $\theta$ for an ideal Gaussian beam is just given by $w_0 \cdot \theta = \frac{\lambda}{\pi}$. [4]

**A4.** A continuous-wave (cw) Nd:YAG laser emits 100mW at a wavelength of 1.064 $\mu$m. The laser has a 10 cm-long two-mirror cavity with mirror reflectivities of $R_1 = 1$ and $R_2 = 0.9$. Calculate

(i) The stimulated emission rate in s$^{-1}$.

(ii) The number of photons inside the cavity.

[6]

## Section B

**B1.** A high-reflectivity mirror can be made using layers of alternating high and low refractive index materials, each with an optical thickness of $\frac{1}{4}\lambda$, deposited on a substrate.

(a) Describe the physical mechanism responsible for the high reflectivity of the stack. [4]

(b) Summarise briefly the advantages of the use of multilayer stack mirrors in laser cavities over mirrors based on a single metal layer. [3]

(c) A Nd:YAG laser running at a wavelength $\lambda = 1.06\,\mu\mathrm{m}$ has a cavity which requires two mirrors: a highly-reflecting mirror and an output coupling mirror. The mirrors are to be made on a glass substrate with refractive index $n_T = 1.5$, using pairs of layers of $MgF_2$, $n_1 = 1.38$, and $ZrO_2$, $n_2 = 2.1$. If the reflectivity of the mirror has to be maximum at a wavelength $\lambda = 1.06\,\mu\mathrm{m}$, what is the thickness of each layer in a single pair? [2]

(d) The high reflector is required to have reflectivity $R = 0.99$, and the output coupler is required to have $R = 0.6$. Show that the number of pairs of $MgF_2/ZrO_2$ layers necessary to produce the required reflectivity are 7 and 2 respectively. [7]

(e) Using a similar Nd:YAG crystal, a *monolithic* laser cavity is to be produced, where the mirrors are deposited directly onto the ends of the crystal instead of being deposited onto a substrate. Show that to maintain a reflectivity close to $R = 0.6$, the output coupler in this case will need at least one extra pair of layers. [4]

**B2.** The *complex curvature parameter,* $q$ of a Gaussian beam is given by

$$\frac{1}{q(z)} = \frac{1}{R(z)} - \frac{i\lambda}{\pi w^2(z)}$$

where $R$ is the radius of curvature of the beam, $\lambda$ is the wavelength, and $w$ is the spot size.

(a) What is the physical interpretation of the position in space where the complex curvature parameter of a beam is purely imaginary? [2]

(b) The complex curvature parameter after propagation through an optical system is given by

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D}$$

where $A, B, C$ and $D$ are the coefficients of the optical system's $ABCD$ matrix. By considering the propagation of a beam with complex curvature parameter $q$ over a single round trip $ABCD$ matrix, show that a stable cavity must have $(D + A)^2 < 4$. [4]

(c) An optical cavity is formed of a plane mirror and a concave spherical mirror with radius of curvature $R$, separated by a distance $l$. Show that the round-trip $ABCD$ matrix is given by

$$\begin{pmatrix} 1 - \frac{4l}{R} & 2l \\ -\frac{2}{R} & 1 \end{pmatrix}.$$

[4]

(d) Hence show that the cavity is stable when it has lengths between $l = 0$ and $l = R$. [2]

(e) Cavities with one plane mirror have modes similar to the equivalent *symmetric* cavity with two mirrors of curvature $R$. For $l = R/2$ and $l = R$ in the asymmetric cavity described in part (c), what are the equivalent symmetric cavities? [2]

(f) By considering the properties of the equivalent symmetric cavities, describe the change in the

(i) spot size at each of the cavity mirrors

(ii) output beam divergence

between cavity lengths of $l = R/2$ and $l = R$. [6]

**B3.** The rate equations for the population inversion density, $N_2$, and the total photon number, $\phi$, in a simple 4-level laser are:

$$\frac{dN_2}{dt} = \Gamma - \frac{(c/n)\sigma}{V}\phi N_2 - \frac{N_2}{\tau}$$

$$\frac{d\phi}{dt} = V_a \frac{(c/n)\sigma}{V}\phi N_2 - \frac{\phi}{\tau_c}$$

where $\Gamma$ is the pumping rate per unit volume into the upper laser level, $\sigma$ is the emission cross-section, $n$ is the refractive index of the gain medium, $V$ is the cavity mode volume, $V_a$ is the volume of the mode in the gain medium, $\tau$ is the upper state lifetime, and $\tau_c$ is the cavity lifetime.

(a) Describe the physical origin of each of the terms on the right-hand side of these two equations. [5]

(b) Show that, in the steady state, the population inversion $N_2$ is related to the laser intensity $I$ by

$$N_2 = \frac{N_0}{1 + \frac{2I}{I_{sat}}}$$

where $N_0 = \Gamma\tau$, and the saturation intensity $I_{sat} = \hbar\omega/\sigma\tau$, where $\omega$ is the frequency of the laser transition. What is the physical interpretation of the saturation intensity of a laser transition? [5]

(c) For an optically pumped laser, explain why the pump laser power $P_{pump}$ can be written as $P_{pump} = \Gamma\ V_a\ \hbar\omega_p$, where $\omega_p$ is the frequency of the pump laser. [1]

(d) In steady state operation, the laser output power $P_{laser}$ is given by:

$$P_{laser} = \left(\frac{T_2}{T_2 + A}\right)\left(\frac{\omega}{\omega_p}\right)\left[P_{pump} - P_{threshold}\right]$$

where $A$ is the loss in the cavity that is not associated with output coupling, $P_{pump}$ is the incident pump power, and $P_{threshold}$ is the incident pump power at threshold. How would you expect the slope efficiency and the threshold power to vary as $A$ is increased? [2]

(e) What is the maximum possible slope efficiency for a Nd:YLF laser pumped at 805 nm, and oscillating at 1.047 $\mu$m? [2]

(f) Sketch the variation of laser output power as the transmission of the output coupler is increased from zero. Briefly describe physically why the graph has this form. [5]

**B4.** (a) For Q-switched and mode locked lasers, make a comparison of

(i) Pulse length

(ii) Pulse energy

(iii) Pulse peak power

for typical laser systems. [3]

(b) Compare the variation in the population inversion with time inside a Q-switched laser and a mode locked laser. [2]

(c) Describe physically why a very short, unchirped pulse of light travelling through a block of glass emerges with a longer pulse length. How does the instantaneous frequency vary with time after the pulse emerges from the block? [4]

(d) A Gaussian pulse propagates through a medium with second order dispersion $k'' = \frac{d^2k}{d\omega^2}$. The field envelope $A(z,t)$ is given at position $z$ by

$$A(z,t) = \frac{A_0}{\sqrt{(z_c + iz)}} \exp\left[-\frac{t^2}{2k''(z_c + iz)}\right]$$

where $t$ is the time in the frame of the moving pulse, and $z_c$ is the chirp length, given by $z_c = \frac{\tau_0^2}{2k''}$. Show that this leads to a variation in the time width of the pulse with distance, $\tau(z)$, given by

$$\tau(z)^2 = \tau_0^2\left[1 + \frac{z^2}{z_c^2}\right].$$

[5]

(e) Give a physical interpretation of the chirp length, $z_c$. [1]

(f) A chirped pulse amplifier (CPA) system uses a length of optical fibre as a pulse stretcher. The amplification system requires that the peak intensity of the pulse be reduced by a factor of 1000 by the stretcher before amplification. What length of fibre will be necessary if the input pulse has a full width at half-maximum (FWHM) of 10 fs? The fibre dispersion $k'' = 26\,\mathrm{fs}^2/\,\mathrm{cm}$. [4]

(g) Give an example of an optical system which could be used to compress the stretched pulse after amplification. [1]

END OF PAPER