SEMESTER 1 EXAMINATION 2014-2015

COSMOLOGY AND THE EARLY UNIVERSE

Duration: 120 MINS (2 hours)

This paper contains 8 questions.

Answer **all** questions in **Section A** and **one** question in **each** of **Section B** and **Section C**.

Each section carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on each.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

The default system of units is SI. You may give expressions in natural units, but you should state when you start to use them and note when you change unit systems. Throughout the paper the scale factor is normalized in such a way that at the present time $a_0 = 1$. The Friedmann equation is

$$H^2 = \frac{8\,\pi\,G}{3\,c^2}\,\varepsilon - \frac{k\,c^2}{a^2\,R_0^2}\,.$$

Section A

A1. Define and write down the expression of the Hubble distance at the present time. Give a numerical estimate (1 significant figure) using $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. [5]

A2. Consider an empty Universe within Friedmann cosmological models. Is it an open, flat or closed Universe? Motivate your answer. Using $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and using $1\,\mathrm{Mpc} = 3 \times 10^{19}\,\mathrm{Km}$, calculate the age of the Universe, properly defined, in years (2 significant figures). [6]

A3. Consider an inflationary stage occurring in the time interval $[t_\mathrm{i}, t_\mathrm{f}]$, with $t_\mathrm{f} - t_\mathrm{i} = 10^{-32}\,\mathrm{s}$ and number of e-folds $N = 100$. What is the value of the expansion rate H_i during inflation (in s^{-1})? [5]

A4. Explain what is meant by Big Bang Nucleosynthesis (1 sentence). Consider T_in, the initial (and highest) temperature during the expansion of the Universe within the ΛCDM model. The value of $k_B\,T_\mathrm{in}$ has to be assumed much greater than a certain value in order to reproduce correctly the measured primordial abundances. What is this value (1 significant figure) and what is its physical significance? [4]

Section B

B1. (a) Consider the Friedmann equation. What is the name and the physical meaning of a, H, ε and k (no more than 1 sentence for each quantity)? [4]

(b) What values can k take and to what kinds of Universe do they respectively correspond? Consider a Friedmann cosmological model with an admixture of radiation, matter and non-vanishing cosmological constant. Is the value of k unambiguously determining the fate of the Universe expansion? Motivate your answer. [5]

(c) What is the physical meaning of the critical energy density ε_{c}? Derive an expression for ε_c starting from the Friedmann equation. How is the energy density parameter Ω defined? [4]

(d) Show that the Friedmann equation can be recast in terms of Ω_0 and H_0 as

$$\dot{a}^2(t) = H_0^2\,\Omega_0\,a^2(t)\,\frac{\varepsilon(t)}{\varepsilon_0} + H_0^2\,(1-\Omega_0)\,.$$ [7]

TURN OVER

B2. (a) Write down the cosmological fluid equation in any equivalent form you prefer. Show that for a fluid with equation of state $p = w\,\varepsilon$, where $w = \mathrm{const}$, the total energy density depends on the scale factor a as

$$\varepsilon(a) = \frac{\varepsilon_0}{a^{3\,(1+w)}}\,.$$

[4]

(b) Derive the acceleration equation combining the Friedmann equation with the fluid equation. [5]

(c) Consider now *flat* one-fluid Friedmann cosmological models with an equation of state $p = w\,\varepsilon$ and $w = \mathrm{const} > -1$.

- Define the age of the Universe t_0 and derive an expression for it in terms of w and H_0.
- Derive an expression for the scale factor $a(t)$ in terms of t_0, H_0 and w.
- What is that particular value of w such that $t_0 = H_0^{-1}$?
- Which is the other (non-flat) Friedmann cosmological model with $t_0 = H_0^{-1}$?

[11]

Section C

C1. (a) What are the three main observational features of the Cosmic Microwave Background (CMB) radiation spectrum? [5]

(b) Determine (1 significant digit) the value of $k_B\, T_{\rm eq}^{RM}$, where $T_{\rm eq}^{RM}$ is the temperature at the matter-radiation equality time. Use for the radiation energy density parameter at the present time $\Omega_{R,0} \simeq 0.75 \times 10^{-4}$, for the matter energy density parameter $\Omega_{M,0} \simeq 0.32$, for the Boltzmann constant $k_B = 0.86 \times 10^{-4}\, {\rm K\, eV}^{-1}$. [3]

(c) Considering the observational features of the CMB radiation and the properties of photons, explain why the distribution function of relic photons (giving the average occupation number of each quantum state) is very well approximated by the Bose-Einstein distribution,

$$f_{\gamma,0}(p) = \frac{1}{e^{\frac{c\,p}{k_B\,T_0}} - 1}\,.$$

[3]

(d) Derive the number density of relic photons at the present time in terms of the CMB radiation temperature T_0. Calculate its numerical value with two significant figures using for the Boltzmann constant $k_B = 0.86 \times 10^{-4}\, {\rm K\, eV}^{-1}$.

Hint: You might find useful

$$\int_0^\infty dx\, \frac{x^2}{e^x - 1} = 2\,\zeta(3) \simeq 2.4\,.$$

[7]

(e) Derive the distribution of the relic photons back at the matter-radiation decoupling time $t_{\rm dec}^{\rm RM}$. Explain the physical meaning of the result (1 sentence). [2]

TURN OVER

C2. (a) What are the main features of the ΛCDM model? Specify in particular what is the matter-energy budget at the present time, distinguishing the contribution from ordinary baryonic matter and the contribution from Dark Matter. [5]

(b) The cosmological redshifts of supernovae indicate the approximate relation

$$\Omega_{\Lambda,0} \simeq 1.5\,\Omega_{M,0} + 0.25\,.$$

Write down the complementary relation linking $\Omega_{M,0}$ to $\Omega_{\Lambda,0}$ coming from the study of the CMB radiation anisotropies. What is the specific feature in the CMB radiation anisotropies that strongly supports it? Combining the two relations, calculate the values of $\Omega_{\Lambda,0}$ and $\Omega_{M,0}$ (1 significant figure). [5]

(c) Discuss qualitatively which astronomical observations support the existence of Dark Matter. What is the interpretation of the nature of Dark Matter more strongly supported by the current astronomical and cosmological observations? [5]

(d) Most of the visible matter in Galaxies is concentrated within a few kpc from the centre. Show how the Newton's law of gravity implies that the speed of a star at a distance $R \gtrsim 10\,\mathrm{kpc}$, i.e. far from the central bulge, should decrease like $v(R) \propto 1/\sqrt{R}$ in the absence of Dark Matter. [5]

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