Achievement Award: Sample Solutions

These are *model* solutions. Alternative explanations and derivations are of course acceptable provided they are valid.

1. A lunar eclipse occurs when the moon passes into the shadow of the Earth.

(a) Explain why this occurs at full moon and not at new moon.

(b) Explain why we do not observe a full lunar eclipse every full moon. [4]

Answer

Just before the moon passes into the shadow of the earth it is almost collinear with the sun and earth and on the opposite side of the earth from the sun [1]. This means that when viewed from the earth it is fully illuminated (full moon) [1].

The orbit of the moon around the earth is not coplanar with the orbit of the Ear th around the sun [1]. A full lunar eclipse can only occur if full moon coincide s with the intersection of the planes of these two orbits [1].

Nearly all of this can be shown in a sketch. A complete absence of a sketch, which has been requested, even if all explained in text, removes one mark (i.e. maximum 3 without a figure).

2. An ice-cold drink initially has some crushed ice in it. Draw a graph of the temperature of the drink as a function of time as it warms to room temperature. [4]

Answer

The drink will remain at the same temperature for a while until all the ice melts, then warm to room temperature. The figure should show:

1. temperature vs time [1]
2. the initial flat phase [1], indicating that the temperature rises once all the ice has melted [1]
3. the subsequent rise to room temperature [1]

3. Explain how a boat made of steel floats.

A fisherman in a small boat on a lake throws the anchor made of steel overboard. The anchor sinks to the bottom of the lake. Does the level of the water in the lake rise or fall? [7]

Answer

There are several routes to the answer to the first part, including the fact that the boat has a mean density less than that of water. A reasonable physical answer to this is worth [3] marks.

While the anchor is in the boat it displaces a volume of water with a mass equal to its own mass [1]. Since iron has a higher density than water (the anchor sinks rather than floats), the volume of the displaced water is larger than the volume of the anchor [1]. When the anchor is dropped overboard, it displaces a volume of water equal to its own volume, less than it displaces when in the boat [1]. Therefore the water level in the lake falls [1].

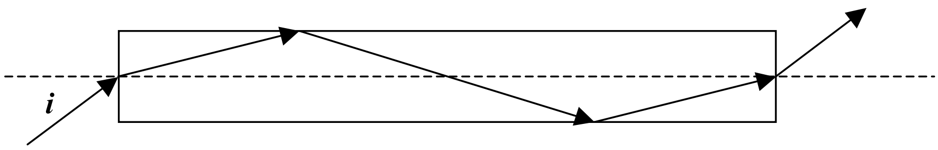
4. I come across a group of training athletes who are running at a constant speed of $15\,\mathrm{km\,h^{-1}}$ around a 10 km track, such that they are evenly spaced with a separation of 500 m. If I run a single lap at a speed of $10\,\mathrm{km\,h^{-1}}$, how many athletes will pass me: (a) if I run in the same direction; (b) if I run in the opposite direction? [6]

Answer

The relative velocity between me and another running going in the same direction is $5\,\mathrm{km\,h^{-1}}$ [1]. So, I see a runner coming up behind me approaching me with $5\,\mathrm{km\,h^{-1}}$. The distance between us is initially 500 m and so it takes $0.5\,\mathrm{km}/5\,\mathrm{km\,h^{-1}} = 0.1\,\mathrm{h} = 6\mathrm{min}$ until I am overtaken [1]. Since I run at $10\,\mathrm{km\,h^{-1}}$, it takes me $1\,\mathrm{h} = 60\mathrm{min}$ to complete the round. Therefore I am overtaken by 10 runners [1]. Note that it does not matter what the starting configuration is.

In the other direction the relative velocity is $25\,\mathrm{km\,h^{-1}}$ [1] and so the time between two encounters is $0.5\,\mathrm{km}/25\,\mathrm{km\,h^{-1}} = 0.02\,\mathrm{h} = 1.2\mathrm{min}$ [1]. This implies that I meet $60\mathrm{min}/1.2\mathrm{min} = 50$ runners going the other way [1]. However, since only $10\,\mathrm{km}/0.5\,\mathrm{km} = 20$ runners fit onto the course, I encounter only those 20 individual runners, some of them 3 times.

5. A laser beam is incident at the midpoint of one side of a block of glass of length 12 cm and width 2 cm at an angle $i = 27°$. After undergoing total internal reflection from opposite sides, as shown in the diagram below, it emerges from the point opposite the point of entry. Calculate the refractive index of the glass.



Why does no light escape from the long sides of the block? [8]

Answer

The key here is to realise that it is a geometrical problem to determine the angle of refraction for the ray entering the block. Given the angles of incidence and reflection, it is then a matter of applying Snell's law.

From the diagram, one half of the block's width is crossed in one quarter of its length [1]. So the internal angle, j, is given by

$$\tan j = \frac{2\,\mathrm{cm}/2}{12\,\mathrm{cm}/4} = \frac{1}{3}, \qquad j = 18.4° \text{ [1]}.$$

Snell's law reads

$$n_a \sin i = n_g \sin j \text{ [1]},$$

where n_a and n_g are the refractive indices of the air and the glass respectively. Taking the refractive index of air to be 1 (a good approximation) [1], leads to $n_g = 1.44$ [1].

Now consider the beam hitting the long edges of the block. The angle of incidence is arctan(3) or 71.6° [1]. The angle of refraction, k, for light escaping from the glass would then be given by

$$\sin k = n_g \sin(71.6^\circ) = 1.36 \text{ [1]}$$

which has no solutions for real k [1]. Thus no light escapes. This is the phenomenon of total internal reflection.

6. A comet of mass m is travelling radially away from the Sun at speed v. The Sun has mass M. At distance r from the Sun the gravitational potential energy of the comet is

$$E_{\text{grav}} = -\frac{GmM}{r},$$

where G is the gravitational constant.

Explain the meaning of the minus sign in this formula.

The total energy of the comet, E_{tot}, is given by the sum of its potential and kinetic energies. What condition must E_{tot} meet so that the comet cannot leave the gravitational field of the Sun, that is to prevent $r \to \infty$? [5]

Answer

[Since the gravitational interaction weakens with distance, we use the standard convention that the gravitational potential energy is zero at infinite separation.]

The gravitational force is attractive so the potential energy of the Sun-comet pair must increase as their separation increases. This necessitates a minus sign in the expression for the potential energy. [1]

While moving away from the Sun, the comet is constantly decelerated by the gravitational force and kinetic energy is converted to potential energy. The total energy is

$$E_{\text{tot}} = \frac{1}{2}mv^2 - \frac{GmM}{r}. \text{ [2]}$$

If the comet cannot escape from the Sun, then its speed v must become zero at some finite distance, say d. In this case, the energy equation becomes

$$E_{\text{tot}} = -\frac{GmM}{d} \text{ [1]}$$

and we can only solve this for positive d if $E_{\text{tot}} < 0$ [1].

7. Estimate the number of grains of rice in a 1 kg bag of rice. [5]

Answer

The exact numbers in this estimate are not crucial, but the reasoning is important.

Consider a grain of rice to be a cylinder of diameter 2 mm and length 5 mm [1]. Its volume is then

$$\pi \times (10^{-3}\,\mathrm{m})^2 \times 5 \times 10^{-3}\,\mathrm{m} = 1.6 \times 10^{-8}\,\mathrm{m}^3. \text{ [1]}$$

Raw rice (before cooking) just sinks in water, so we approximate its density by, say, 1.25 times the density of water, $1000\,\mathrm{kg\,m^{-3}}$ [1]. The mass of a grain is then

$$1.6 \times 10^{-8}\,\mathrm{m}^3 \times 1.25 \times 1000\,\mathrm{kg\,m^{-3}} \approx 2 \times 10^{-5}\,\mathrm{kg}, \text{ [1]}$$

or 0.02 g. The number of grains in 1 kg of rice is then about 50 000 [1].

8. A truck of mass M is travelling in a straight line with a constant velocity v. A bag of sand of mass m is dropped from a small height (with no horizontal component of velocity) on to the back of the truck. What is the velocity of the truck after the bag has been dropped? Assume that the bag does not slide on the back of the truck.

Show that the kinetic energy of the truck plus the bag, after the bag has fallen on the truck, is smaller than the kinetic energy before the bag is dropped, and explain where the energy has gone. [5]

Answer

Momentum in the horizontal direction is conserved [1] (at least for the short duration of the impact of the bag of sand) and so

$$Mv = (m + M)\,v', \text{ [1]}$$

where v' is the velocity of the truck plus the bag. Solving for v' we find

$$v' = \frac{M}{M+m} v.$$

The kinetic energy of the truck before the bag is dropped is

$$E = \frac{1}{2} M v^2.$$

Afterwards the kinetic energy is

$$E' = \frac{1}{2}(M+m)(v')^2 = \frac{1}{2} M v^2 \frac{M}{M+m}. \text{ [1]}$$

Since $M + m > M$, the kinetic energy has decreased [1]. The energy is converted into heat by the friction which prevents the bag from sliding off the truck [1].

End of paper